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PRINCIPLES
OF
ELECTRICAL MEASUREMENTS

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BY

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PREFACE

This book is written for the instruction of those who are beginning their course in Electrical Engineering, or who desire a more complete understanding of this branch of Physics than is afforded in most elementary manuals, and as far as possible the first consideration has been the requirements of such readers. It is the result of ten years of teaching the subject in the University of Michigan and as now presented it meets the requirements for a class book as well as a laboratory manual. The aim has been to lead the student to learn the facts from his own observations, and direct information is often replaced by a suggestion how the information can be obtained. This leads to independent investigation and does not dull the keen pleasure of discovery by knowing the result before the experiment is tried.

The book is arranged on the progressive system. The simpler and more fundamental parts of the subject are taken up in the first chapters, and in the first part of each chapter, while the more difficult measurements and the methods involving more extended knowledge are reserved until the student has attained greater proficiency. For example, Chapter I shows how to measure current, E.M.F., resistance, and power, by ammeter and voltmeter methods. For an elementary course in Electrical Measurements nothing could be better than this series of simple experiments, well understood. They bring out the fundamental relations with a minimum of apparatus to confuse the mind; and they are not out of place at the beginning of a more extended course which contemplates using the entire book.

In deducing the formulas it has not been considered sufficient merely to write down the equations, but in each case they are

worked out logically from the fundamental relations, and the reader is led to use his reason rather than his memory. Thus in Chapter VII, considerable use is made of Kirchhoff's second law, not so much on account of the importance of the law itself but because it enables one to write down in the form of equations many of the relations that are found to exist when the experiment is performed in the laboratory. There is only enough repetition to make the work intelligible at whatever point it may be taken up, as must be necessary in a large class where all cannot take the experiments in the order in which they are given in the book.

As the subject is carried further more theoretical treatment is required, until in the chapters on magnetism the relations between the quantities involved had to be very completely worked out from the fundamental definitions because this part of the subject is lightly passed over by most writers. In fact, the ideas of many students regarding B and H are not only vague, but often are erroneous, and it has been my privilege to lead several hundred such students to a clear and useful understanding of this interesting part of Physics. If this book shall be the means of enlarging the circle of those who are thus helped to think for themselves I shall feel that it has not been written in vain.

I desire to express my thanks to Dean Karl E. Guthe, who has kindly read the entire proof, and to Assistant Professor Neil H. Williams, who has read the chapters on magnetism and alternating currents; also to my wife, Frances Berry Smith, for assistance in the preparation of the index.

Arthur Whitmore Smith.

University of Michigan,
Sept., 1914.

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PRINCIPLES OF ELECTRICAL MEASUREMENTS.

INTRODUCTION

UNITS AND DEFINITIONS

1. C.G.S. Systems.—In the measurement of electrical quantities there are two distinct systems of units that may be used. In the electrostatic system the fundamental unit is determined by the repulsion of two similar charges of electricity. In the electromagnetic system it is the repulsion of two similar magnetic poles that determines the values of the units.

Each of these systems is properly called an "absolute," or a "C.G.S." system, but all of the common units such as ohm, ampere, volt, etc., are based solely upon the electromagnetic C.G.S. system of units.

2. Unit Magnetic Pole.—When two bars of steel are magnetized it is found that one end of one will attract one end of the other, while the other end is repelled by the same end of the first one. Particularly is this the case if the bars are long thin needles with ball ends.

The laws relating to this attraction and repulsion have been carefully studied, and while the resultant effect must actually be the sum of all the separate effects of each part of one magnet upon each part of the other, yet for *purposes of computation* the correct result is obtained quite simply if the action is considered as due to a "pole" concentrated at a point not far from the center of the ball on the end of a ball-ended magnet. As the result of careful measurements, Coulomb found that the force of repulsion between two similar poles is given by the expression,

$$F = k\,\frac{mm'}{r^2}\ \text{dynes},$$

where r is the distance in centimeters between the two points

at which are concentrated the poles of strength m and m' respectively.

Faraday further showed that if the stationary medium between these poles has a permeability μ, the force is then μ times smaller. This gives,

$$F = k \frac{mm'}{\mu r^2} \text{ dynes,}$$

as the complete expression for the force between two poles; k is the proportionality factor to give the result in dynes.

Apparently the most rational unit with which to measure pole strength would be one of such a value as to make $k = 1$.

Definition.—A unit magnetic pole is one of such strength that when placed 1 *cm. from an equal pole, in a vacuum, it will be repelled with a force of* 1 *dyne.*

3. Unit Magnetic Field.—When a magnetic pole is brought near a magnet or an electric current it experiences certain forces urging it to move in a definite direction. A force which thus acts upon a magnetic pole is called a *magnetic force;* and the region in which magnetic force is manifested is called a *magnetic field.*

The force exerted upon a magnetic pole depends quite as much upon the strength of the pole as upon the field, being given by the relation,

$$F = Hm, \quad \text{or} \quad H = \frac{F}{m}.$$

The intensity of the magnetic force in a magnetic field, or more briefly, the *field intensity,* is thus measured by the *force per unit pole* which is exerted upon a magnetic pole. The name "gauss" for unit magnetic field was adopted by the International Electrical Congress at Paris in 1900.[1]

Definition.—A gauss is the intensity of a magnetic field in which a magnetic pole experiences a force of 1 *dyne per unit pole.*

[1] See Elec. Rev., Vol. 47, p. 441, 1900.

4. Magnetic Effect of an Electric Current.—When the poles of a battery are joined by a wire various things happen. Among others, the wire becomes warm to a greater or less degree and in some cases may even be melted. More important still the space surrounding the wire becomes a magnetic field in which a small magnetized needle always tends to set itself at right angles to the wire. A careful examination of this field shows that the positive pole of such a needle is urged along a plane curve encircling the wire, which for the case of a straight wire becomes a circle whose center is at the center of the wire and whose plane is normal to the wire.

When these phenomena are observed it is said that an electric current is flowing along the wire. The positive direction of this current is taken, by convention, as bearing the same relation to the direction of the force acting upon the positive pole as the translation of a right-handed screw bears to the direction of its rotation.

It is evident that considering the magnetic needle as a whole, there is no resultant force tending to move it in any direction, for whatever forces act upon one pole, they are balanced by equal and opposite forces acting upon the other pole. Thus the negative pole of the magnet is urged around the current in the opposite direction, and therefore no useful work can be obtained by carrying the magnet as a whole around the current. For while the positive pole will be urged along with a force proportional to the intensity of the magnetic field, it will be necessary to supply an equal force to carry the negative pole along the same direction. But for purposes of definition we can consider these forces separately.

The existence of these forces means that work is done whenever the poles of the magnet are moved, positive work being done by one pole and negative work by the other. For simplicity let us consider only the positive pole. The amount of work required to carry such a pole once around the current and back to the starting point against the forces of the magnetic field depends upon the strength of the pole and the value

of the current. When the work is measured in ergs this leads at once to the definition of the electromagnetic C.G.S. unit of current.

Definition.—Unit current is flowing in a circuit when it requires 4π ergs per unit pole to carry a magnetic pole once around the current.

This is the fundamental definition of unit current, and in addition to being brief it will be found most useful. One advantage of this definition is that it defines the value of a real current, flowing through an actual circuit, in terms of a magnetic pole of some possible size. (See Chapters X and XI for further applications of this definition.)

The following corollary must also be true.

Corollary I.—The work required to carry a magnetic pole round any path not enclosing a current is zero.

When a current flows through a circular loop of wire, the only point that is symmetrically located with respect to the current is the center of the circle. The intensity of the magnetic field at this point depends not only upon the value of the current, but also upon the length of wire in the loop and its distance from the center. This leads to

Corollary II.—A current of 1 C.G.S. unit flowing through a wire bent to the arc of a circle of 1 cm. radius will produce at the center a field intensity of 1 gauss for each centimeter length of wire.

The 4π enters the definition above in order to give to this corollary the apparent simplicity with which it is stated. In some respects it would have been simpler if the unit current could have been defined 4π times smaller.

For proof of Corollary II. see Chapter XI.

5. Unit Quantity.—The definition of unit quantity of electricity follows at once from the definition of unit current.

Definition.—Unit quantity of electricity is the quantity transferred by unit current in one second.

This unit quantity is 3×10^{10} times larger than the electrostatic unit of quantity mentioned at the beginning of page 1.

6. Resistance.—When an electric current is flowing through a circuit it requires a continual expenditure of energy to maintain the current. This energy appears in the form of heat in the conductor carrying the current. The amount of heat thus generated depends upon the amount of current flowing through the conductor, the time that it flows, and upon a property of the conductor called its resistance. By using different currents and different conductors it has been found that the amount of heat produced in a given conductor is proportional to the square of the current, so that, if W denotes this amount of heat energy measured in ergs,

$$W = R I^2 t \text{ ergs},$$

where R denotes the resistance of the conductor and t the time that the current I is flowing. Of course the resistance must be measured in units of such magnitude as will make this expression a true equation. This leads at once to the following definition:

Definition.—Unit resistance is that resistance in which one erg of heat is produced each second by the passage of unit current.

7. Difference of Potential, Electromotive Force, Fall of Potential.—The difference of potential between two points is that difference in condition which produces a current from one point to the other as soon as they are connected by a conductor. Every battery or other electric generator possesses a certain power of maintaining a difference of potential between its terminals, and therefore a power of driving a continuous current. This difference of potential produced by a cell or other generator, and which may be considered as the cause of the current, is called electromotive force. It must be remembered that this quantity is not a force at all, and in order to avoid using the word "force" it is commonly called E.M.F.

When a current flows through a conductor there is a difference of potential between any two points on the conductor. This difference of potential is greater the further apart the points

are taken and as the change is gradual it is usually called a "fall of potential." It can always be expressed by the formula $R\,I$, where R is the resistance of the conductor, or conductors, under consideration.

This apparent duplication of names may at first appear unnecessary, but the corresponding ideas are quite distinct and the correct use of the proper term will add conciseness to one's thinking and speaking. Thus we have the E.M.F. of a battery; the fall of potential along a conductor; and the more general and broader term, difference of potential, which includes both of the above as well as some others for which no special names are used.

8. Unit Potential Difference.—Having now defined the unit of current and the unit of resistance, the value to be chosen for the unit difference of potential follows from Ohm's law.[1]

Definition.—Unit potential difference is the difference of potential over unit resistance when carrying unit current.

Since $E = RI$, and $Q = It$, from the preceding definitions, the amount of energy expended in a conductor by an electric current may be expressed as

$$W = R\,I^2\,t = E\,Q,$$

where Q denotes the total quantity of electricity that has passed through the conductor. This gives another way of stating the value of unit potential difference, viz.,

Corollary.—Unit difference of potential is that difference of potential through which one erg can raise a unit quantity of electricity.

9. The Practical Units. *The Ohm.*—The unit of resistance just defined is inconveniently small (because an erg is so small) and therefore for practical use a multiple unit, 1,000,000,000 *times as large,* is used. This larger unit is called an *ohm.*

The Ampere.—In the same way, the unit of current defined above turns out to be too large for convenience in practical measurements, and therefore the *ampere* is defined as being *one-tenth of the C.G.S. unit.*

[1] See page 14.

The Volt.—Having now defined the ampere and the ohm, it follows that the practical unit for difference of potential is *the difference of potential that steadily applied to a conductor whose resistance is* 1 *ohm will produce a current of* 1 *ampere.* This unit is called a *volt.* Evidently it is 100,000,000 C.G.S. units.

The Coulomb.—Having the ampere for the practical unit of current it follows that the corresponding unit of quantity is the quantity transferred by one ampere in one second. This unit is called a *coulomb.*

The Watt.—The power expended in maintaining one ampere under a potential difference of one volt is called a *watt.*

The Joule.—The work done each second by one watt is called a *joule.* This is the work that will transfer one coulomb through a potential difference of one volt.

10. Concrete Examples of these Units.—These definitions of the electrical units do not furnish, directly, any standards for use in making actual measurements. Therefore the most careful investigations have been made to determine the tangible values of these units as defined above, in order to express them in terms of definite concrete quantities. From the very nature of the case, such determinations never can express the exact values of the units, but they have been determined much closer than is ever required in ordinary measurements. At a conference of scientific delegates, which met in London, Oct. 12, 1908, the following resolutions were adopted. These resolutions state the concrete values of the fundamental units in accordance with the best and latest opinion of the scientific men of the world.

11. The Conference on Electrical Units and Standards. London, 1908.

RESOLUTIONS

I. The Conference agrees that as heretofore the magnitudes of the fundamental electric units shall be determined on the

[1] Elec. Review, vol. 63, 1908, p. 738.

electromagnetic system of measurement with reference to the centimeter as the unit of length, the gram as the unit of mass, and the second as the unit of time.

These fundamental units are (1) the Ohm, the unit of electric resistance which has the value of 1,000,000,000 in terms of the centimeter and second; (2) the Ampere, the unit of electric current which has the value one-tenth (0.1) in terms of the centimeter, gram and second; (3) the Volt, the unit of electromotive force which has the value 100,000,000 in terms of the centimeter, the gram, and the second; (4) the Watt, the Unit of Power, which has the value 10,000,000 in terms of the centimeter, the gram, and the second.

II. As a system of units representing the above and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the Conference recommends the adoption of the International Ohm, the International Ampere, and the International Volt, defined according to the following definitions.

III. The Ohm is the first Primary Unit.

IV. The International Ohm is defined as the resistance of a specified column of mercury.

V. The International Ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm.

To determine the resistance of a column of mercury in terms of the International Ohm, the procedure to be followed shall be that set out in specification I, attached[1] to these resolutions.

VI. The Ampere is the second Primary Unit.

VII. The International Ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with the specification II, attached[1] to these resolutions, deposits silver at the rate of 0.00111800 of a gram per second.

VIII. The International Volt is the electrical pressure

[1] See Elec. Review, vol. 63. p. 738.

which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere.

IX. The International Watt is the energy expended per second by an unvarying electric current of one International Ampere under an electric pressure of one International Volt.

The Conference recommends the use of the Weston Normal Cell as a convenient method of measuring both electromotive force and current, and when set up under the conditions specified in schedule C may be taken, provisionally, as having, at a temperature of 20° C., an E. M. F. of 1.0184 volts. (1.0183 since Jan. 1, 1911).

In cases in which it is not desired to set up the Standards provided in the resolutions above, the Conference recommends the following as working methods for the realization of the International Ohm, the Ampere and the Volt.

1. *For the International Ohm.*

The use of copies, constructed of suitable material and of suitable form and verified from time to time, of the International Ohm, its multiples and submultiples.

2. *For the International Ampere.*

(*a*) The measurement of current by the aid of a current balance standardized by comparison with a silver voltameter.

(*b*) The use of a Weston Normal Cell whose electromotive force has been determined in terms of the International Ohm and International Ampere, and of a resistance of known value in International Ohms.

3. *For the International Volt.*

(*a*) A comparison with the difference of electrical potential between the ends of a coil of resistance of known value in International Ohms, when carrying a current of known value in International Amperes.

(*b*) The use of a Weston Normal Cell whose electromotive force has been determined in terms of the International Ohm and the International Ampere.

CHAPTER I

AMMETER AND VOLTMETER METHODS

12. Laws of Electric Currents—Use of an Ammeter.—The Weston ammeter is a good and accurate instrument for the measurement of electric current. It is a very delicate and sensitive instrument and must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the instrument is not burned out thereby.

When it is desired to use the ammeter for the measurement of current it is connected in series with the rest of the circuit, and therefore the entire current passes through the instrument. Great care should always be exercised never to allow a larger current to flow through an ammeter than it is intended to carry. It is always best to have a key in the circuit and while keeping the eye on the needle of the ammeter tap the key gently, thus closing it for a fraction of a second only. If the needle does not move very far the key can be held down for a longer time, and if it is now seen that the needle will remain on the scale the key can be held down until the needle comes to rest. Back of the needle is a strip of mirror, and by placing the eye in such a position that the image of the needle is hidden by the needle itself, the error due to parallax in reading the scale can be avoided.

The scales of these instruments are graduated to read the current directly in amperes. Sometimes the pointer does not

stand at the zero of the scale when no curent is flowing. When this is the case the position of rest should be carefully noted and the observed reading corrected accordingly.

For this exercise join a dry cell, a coil of several ohms resistance, a key, and the ammeter in series, that is, one after the other to form a single and continuous circuit. The current

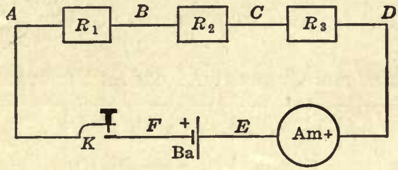


FIG. 1.—Resistances in series.

flows from the positive, or carbon, pole of the cell and should enter the ammeter at the post marked +. Measure and record the value of the current at different points along this circuit, to determine whether the current has the same value throughout its path or whether it is smaller after passing through the resistances. Next add the remaining coils to the circuit,

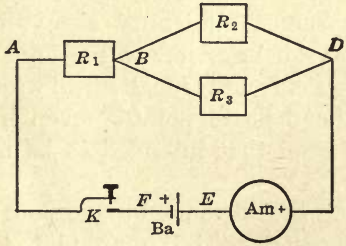


FIG. 2.—Resistances R_3 and R_2 in parallel.

keeping them all in series, and note the value of the current at the same points as before. State in your own words the effect of adding resistance to the circuit.

Remove one of the coils and connect it in parallel with one of those still remaining in the circuit, i.e., so that the current in the main circuit will divide, a part going through each of the

two coils in parallel. Measure the part of the current through each coil: also the main current. Could you have foretold the value of the latter without measuring it?

This exercise should show that the only thing in common to coils in *series* is the value of the *current* passing through them. For this reason an ammeter is always joined in series with the circuit in which the value of the current is desired. Record the data in a neat tabular form similar to the following:

To Measure the Current Along an Electric Circuit

Ammeter zero	Ammeter reading	Current	Point at which current is measured

13. Fall of Potential in Electric Circuit—Use of a Voltmeter.— The Weston voltmeter is a good and accurate instrument for the measurement of electromotive forces. It is a very delicate and sensitive instrument and must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the E.M.F. is not much larger than that intended to be measured by the instrument.

Since a voltmeter is always used to measure the difference of potential between two points it is not put into the circuit like an ammeter, but the two binding posts of the voltmeter are connected directly to the two points whose difference of potential is desired. The voltmeter thus forms a shunt circuit between the two points. There is a current through the voltmeter proportional to the difference of potential between the two points to which it is joined. This current passing through the movable coil of the instrument deflects the pointer over

the scale, but the latter is graduated to read, not the current, but the number of volts between the two binding posts of the voltmeter. In some instruments there is a strip of mirror placed below the needle. When the eye is so placed that the image of the needle is hidden by the needle itself, the reading can be taken without the error due to parallax. Sometimes the pointer does not stand at the zero of the scale when there is no current through the instrument. When this is the case the position of rest should be carefully noted and the observed reading corrected accordingly.

For this exercise join a cell, two coils of several ohms resistance, and a key, in series. With the voltmeter measure the fall of potential over each coil, also over both together. Add a third coil and measure the fall of potential over each; also over all. Note where these last readings are the same as before and where they are different. Add a second cell and repeat the above readings. Note changes.

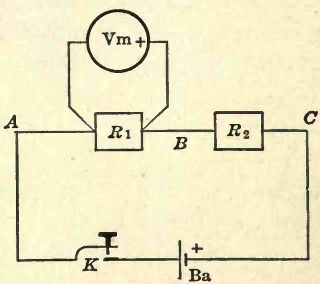


FIG. 3.—Connections for a voltmeter.

Join two of the coils in parallel, thus forming a divided circuit and allowing a part of the current to flow through each branch. Measure the fall of potential over each branch. Add a third coil in parallel with the other two and again measure the fall of potential over each.

This exercise should show, especially in connection with the preceding one, that the only thing in common to several circuits in *parallel* is that each one has the same *fall of potential*. Hence a voltmeter is always joined in parallel with the coil, the fall of potential over which is desired. For the voltmeter indicates the fall of potential over itself; and if it forms one of the parallel circuits, the fall of potential over each one is the same as that indicated by the voltmeter.

Record the data as below:

Fall of Potential in an Electric Circuit

Voltmeter		Fall of potential	Position of voltmeter
Zero	Reading		

14. Ohm's Law.—The current that flows through any conductor is found experimentally to be directly proportional to the potential difference between its terminals. This statement was first formulated in 1827 by Dr. Ohm, and is known as Ohm's Law. It is usually written,

$$V = RI, \text{ or, } I = \frac{V}{R}$$

where V denotes the potential difference over the circuit through which is flowing the current I. The factor R is called the resistance of the conductor, and its value depends only upon the dimensions and material of the wire and its temperature. It is entirely independent of V and I.

This relation holds equally well whether the entire circuit is considered or whether only a portion of such circuit is taken. In the former case the law states that the current through the circuit is equal to the total E.M.F. in the circuit divided by the resistance of the entire circuit, including that of the battery and the connecting wires. When applied to a single conductor, AB, the law states that the current flowing through the conductor is equal to the fall of potential between A and B divided by the resistance AB.

To determine the resistance of a conductor it is then only necessary to measure with an ammeter the current flowing through it, and with a voltmeter measure the difference of potential between its terminals. In case the current is at all

variable the two instruments must be read at the same time, for Ohm's law applies only to simultaneous values of the current and voltage.

15. Measurement of Resistance by Ammeter and Voltmeter. *First Method.*—Join the conductor whose resistance, *R*, is to be measured, in series with an ammeter, *Am*, a key, a battery, and sufficient auxillary resistance to keep the current from being too large. The current should enter the ammeter at the post marked +. Then keeping the eye fixed on the needle of the ammeter, close the key for a fraction of a second. If the deflection is in the right direction, and is not too large, the key can be closed again and the value of the current read from the scale of the ammeter. Should the current be too large, the auxiliary resistance can be increased until the current is reduced to the desired value.

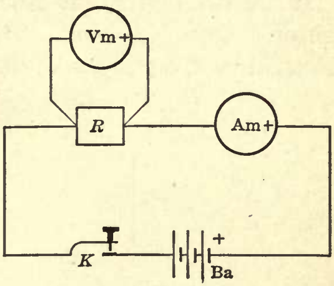


Fig. 4.—To measure a small resistance, *R*.

To measure the fall of potential over the conductor its two terminals are joined to the binding posts of the voltmeter, by means of the additional wires. That terminal of the resistance at which the current enters should be joined to the voltmeter post marked +. Close the key for an instant as before, keeping the eye on the voltmeter needle, and if the deflection is in the right direction and not too large the reading of the voltmeter can be taken.

Now close the key and record simultaneous readings of the ammeter and the voltmeter. Do this three times, changing the current slightly by means of the auxiliary resistance before each set of readings. Compute the resistance of *R* from each set of readings by means of the formula

$$R = \frac{V}{I}$$

where V and I are the voltmeter and ammeter readings, corrected for the zero readings. The mean of these three results will be the approximate value of the resistance.

A more exact value of R can be obtained by correcting the current as measured by the ammeter for the small current, i, which flows through the voltmeter . The current through R is, strictly, not I, but $I-i$. This gives then.

$$R = \frac{V}{I - i}$$

16. Measurement of Resistance by Ammeter and Voltmeter. *Second Method.* This method differs from the First Method by the position of the voltmeter. In the first method

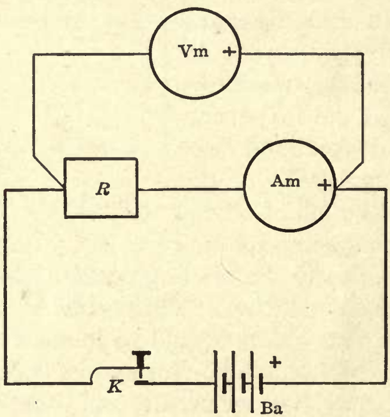


Fig. 5.—To measure a large resistance, R.

the ammeter measured both the current through R and the small current through the voltmeter, and therefore its readings were somewhat too high.

If the connections are made as shown in the figure this error is avoided, as the current now passing through R is strictly the same as that measured by the ammeter. The voltmeter, however, now measures the fall of potential over both R and

the ammeter, and therefore the resistances of both are measured together. The resistance of R is then found by subtracting the resistance of the ammeter from the measured amount. The formula then becomes,

$$R = \frac{V}{I} - A,$$

where A is the resistance of the ammeter.

Measure the resistance of two coils and check results by also measuring them in series and in parallel. The measured resistances should be compared with the values computed from the formula, for series, $R = R_1 + R_2$ and for parallel

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Record the data as follows:

To Measure the Resistance of . . . .

Coil measured	Ammeter reading	I	Voltmeter reading	V	Resistance	
					$\frac{V}{I}$	Corrected

17. To Find the Best Arrangement for Measuring Resistance with an Ammeter and a Voltmeter.—In each of the preceding methods for measuring resistance by means of an ammeter and a voltmeter it is necessary to apply a correction to the observed readings in order to obtain the true value of the resistance being measured. It is the object of this section to inquire under what conditions these corrections are a minimum. In order to compare the two correction terms with each other they will both be expressed in the form of factors by

which the observed values are multiplied to obtain the true values.

In the first method the correction is applied to the ammeter reading, the true current through R being $I - i$. The true value of the resistance is, then,

$$R_1 = \frac{V}{I - i} = \frac{V}{I\left(1 - \frac{i}{I}\right)} = \frac{V}{I\left(1 - \frac{R'}{S}\right)} = \frac{R'}{1 - \frac{R'}{S}} \tag{1}$$

where S is the resistance of the voltmeter, and R' is the uncorrected value of R.

In the second method the value of R is

$$R_2 = \frac{V}{I} - A = R'\left(1 - \frac{A}{R'}\right) \tag{2}$$

where A is the resistance of the ammeter, and R', as before, denotes the ratio of V to I.

The effect of the correction term in (1) is to add something to the measured value of the resistance, while the correction term in (2) reduces it. In using an ammeter and a voltmeter to measure resistance, that arrangement should be chosen, therefore, which involves the smaller correction factor. Comparing these terms it is seen that the former is small, that is, it is near unity, when R' is small; and the second has little effect when R' is large.

The effect of these corrections will be the same when they are equal, that is, when

$$1 - \frac{R'}{S} = 1 - \frac{A}{R'} \tag{3}$$

and this is the case when

$$R' = \sqrt{AS}.$$

For resistances smaller than this the first method has the smaller correction. For larger resistances the second method should be used. Whether the correction is applied or not, it should always be made as small as possible. If the proper method is selected the correction will almost never be as large as 1 per cent.

18. Internal Resistance of a Battery.—The internal resistance of a battery is readily measured by the ammeter and voltmeter method. The cell is joined in series with a suitable resistance, a key, and an ammeter, as shown in Fig. 6. A voltmeter is joined in parallel with the resistance and key. When the key is closed the current drawn from the cell is by Ohm's law.

$$I = \frac{E}{R + r}$$

from which it follows that

$$r = \frac{E - IR}{I} = \frac{E - E'}{I} \tag{A}$$

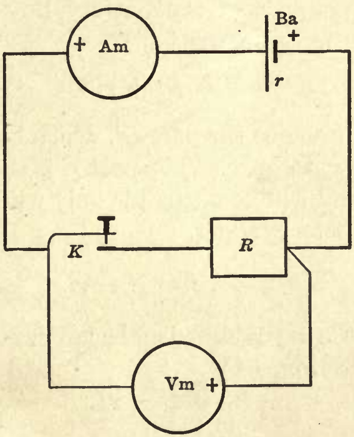


Fig. 6.—To measure the internal resistance of the cell, *Ba*.

E' ($= IR$) being the reading of the voltmeter when the key is closed. The current is measured by the ammeter. When the key is opened the voltmeter is connected directly to the battery and therefore measures E.

If the value of R is known the ammeter can be omitted, and the value of the current determined from the relation $I = E'/R$. Equation (A) then becomes

$$r = R\frac{E - E'}{E'}. \tag{B}$$

As there is apt to be some polarization of the cell it is best to close the key only long enough to obtain simultaneous readings of the ammeter and voltmeter. The value of E is then read immediately upon opening the key.

19. A More Exact Method.—Inasmuch as there is a small current through the cell and voltmeter when the key is open, the foregoing method is subject to a corresponding correction. The value of this current is

$$i = \frac{E}{S + r}$$

where S denotes the resistance of the voltmeter and r includes not only the resistance of the battery but also that of the ammeter. This may be written,

$$E = Si + ri.$$

The voltmeter measures the part Si, which is the fall of potential over its own resistance. The part ri is the fall of potential within the cell and will be negligible only when r is very small. If V is the voltmeter reading,

$$V = Si = E - ri.$$

Similarly when the key is closed and a larger current taken from the cell, we have from (A)

$$E' = E - rI.$$

From these,

$$r = \frac{V - E'}{I - i} \qquad \text{(C)}$$

where i is the voltmeter current with the key open. This reads,

$$\text{Internal Resistance} = \frac{\text{Change in Potential Difference}}{\text{Change in Current}}$$

which is a simple deduction from Ohm's law. E' and I are simultaneous readings of the voltmeter and the ammeter with the key closed. V and i are the corresponding readings with

the key open. Thus the zero readings of both instruments are eliminated. Since the resistance of the ammeter was included in r, it must now be subtracted from the computed result to obtain the resistance of the cell alone. Thus

$$r' = r - A.$$

Record data as follows:

INTERNAL RESISTANCE OF.........CELL

Am. zero = Vm. zero = Am. resistance =

Kind of cell	R	Key closed		Key open			
		E′	I	V	i	r	r'

20. Relation Between Available E.M.F. and Current.—Ohm's law when applied to a complete circuit gives the relation,

$$I = \frac{E}{R + r},$$

or,

$$E = RI + Ir = E' + Ir. \tag{A}$$

The term Ir is the fall of potential over the internal resistance of the cell. RI is the fall of potential over the entire external part of the circuit, and is often denoted by the single symbol E'. Various names have been applied to this term E', such as terminal E.M.F., terminal potential difference, pole potential, available E.M.F., etc. It is that part of the total E.M.F. of the cell that is available for doing useful work. Its value, from eq. A is,

$$E' = E - Ir,$$

and from this it appears that the available E.M.F. is less as the current becomes larger.

A battery is joined to a resistance box and ammeter in series. By using various values of R the current can be varied throughout its possible range, the values being read from the ammeter. A voltmeter joined to the poles of the battery gives the corresponding values of E'. The key should be kept closed as little as possible to avoid unnecessary polarization of the battery. The following values of R will give a good

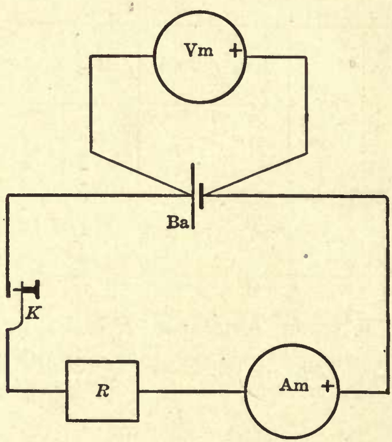


FIG. 7.—To measure the available E.M.F. of the battery *Ba* corresponding to the current through it.

series of readings: 100, 60, 40, 30, 20, 16, 13, 10, 8, 7, 5, 4, 3, 2, 1, ohms.

From the values of E' and I plot a curve. Repeat for one or two different types of cells using at least one cell of low internal resistance. Determine from the curves the maximum current which each cell can furnish. Find one reason why an ammeter should not be joined to the poles of a cell as is done with a voltmeter.

Record the data as follows:

RELATION BETWEEN AVAILABLE E.M.F. AND CURRENT FOR A.....CELL

R	Ammeter		I	Voltmeter		E'
	Zero	Reading		Zero	Reading	

21. Useful Power from a Cell.—Electrical power is measured by the product EI, where I denotes the current and E the fall

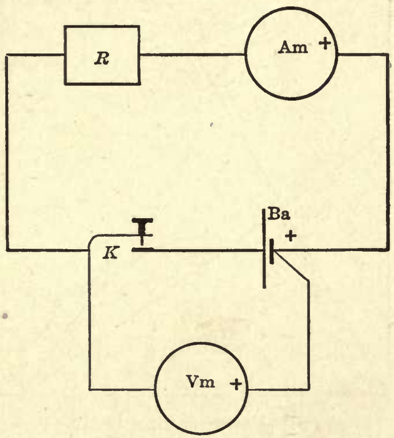


FIG. 8.—Measurement of power.

of potential over the circuit in which the power is being expended. The unit in which power is measured is the watt, one watt being the product of one volt by one ampere.

When a battery is furnishing a current the total power expended is supplied by the chemical reactions within the cell. Part of this power is expended in the external circuit where it may be used in running motors or doing other useful work. The remainder is spent within the cell and only goes to warm-

ing the contents of the battery. In some cases the greater part of the energy is thus wasted within the cell.

The object of this experiment is to measure the power in the external circuit when various currents are flowing. The cell is joined in series with an ammeter and a resistance which will carry the largest current that may be used. A voltmeter measures the fall of potential. Probably there will be found a point beyond which the useful power decreases even through

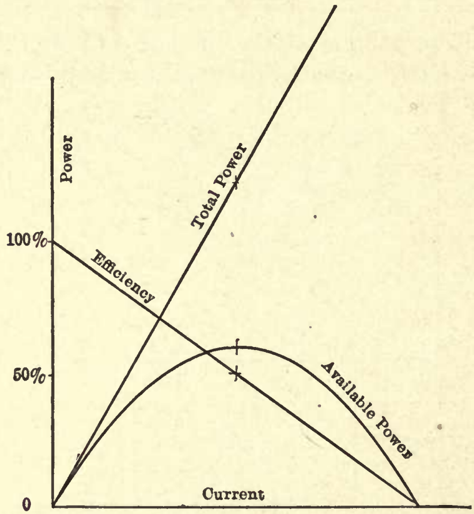


FIG. 9.—Power from a battery.

the current is made larger. Since the current is proportional to the amount of chemicals used up in the cell, it will be well to express the results as a function of the current. This is best done by means of a curve, using values of the current for abscissæ and the corresponding values of power for the ordinates.

The equation of this curve is

$$W = IE' = IE - I^2r.$$

Placing the derivative of W with respect to I equal to zero gives

$$\frac{dW}{dI} = E - 2Ir = 0.$$

as the condition for obtaining the maximum power. This is when $I = E/2r$, or the maximum available power is obtained when the external resistance is equal to the internal resistance of the battery.

Plot also the curve showing the total power, EI, supplied by the cell. The ratio of the ordinates of these two curves, that is, the ratio of the useful power to the total power, gives the efficiency of the cell.

The efficiency can also be plotted as a curve on the same sheet with the other two curves.

Record the data as follows:

USEFUL POWER FROM A .... CELL

R	Ammeter		I	Voltmeter		E'	W
	Zero	Reading		Zero	Reading		

Problem.—How much power is expended in an ammeter of 0.01 ohm resistance if it is connected to a storage battery of ten volts and 0.01 ohm internal resistance? Why is it best not to try the actual experiment in the laboratory?

22. E.M.F. of a Cell by a Voltmeter and an Auxiliary Battery.—When a cell, B, is joined to a voltmeter the reading of the latter does not strictly give the E.M.F. of the cell. This discrepancy is small for cells of low internal resistance, but in many cases it can not be neglected. If the resistance of

the voltmeter is S, and that of the cell is r, then by Ohm's law the small current through the cell and voltmeter is,

$$i = \frac{E}{S + r}.$$

From this the E.M.F. of the cell is,

$$E = Si + ri,$$

where Si is the fall of potential over the external circuit, in this case the voltmeter. Since a voltmeter measures only the fall of potential over its own resistance the reading of the voltmeter is not E, but Si, which is less than E by the amount ri. When r is small this term can be neglected, but not otherwise, unless i can be made very small.

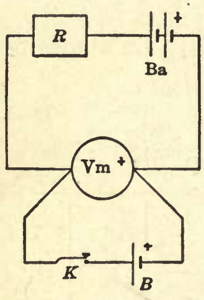


Fig. 10.—To measure the E.M.F. of B.

In the following method no current is taken from the cell and therefore its full E.M.F. can be measured. A second circuit is formed, consisting of the voltmeter, two cells, and a resistance R, which is adjusted until the voltmeter reading with K open is about equal to the E.M.F. of B. There is thus introduced into the former circuit of B an additional difference of potential, viz., SI, the fall of potential over the voltmeter due to the current I of the second circuit. When K is closed the current through the first circuit will now be,

$$i = \frac{E - SI}{S + r},$$

This current i flowing through the voltmeter in addition to the current I, will cause an increased deflection. It is possible to adjust R to such a value that closing K will not affect the reading of the voltmeter; in other words it is possible to make $i = 0$. This means that

$$E = SI,$$

and SI, being the fall of potential over the voltmeter is given directly by the voltmeter reading.

Measure in this way the E.M.F. of several cells and compare the results obtained in each case with the readings of the voltmeter when used alone.

The data may be recorded as follows:

Name of cell	Voltmeter readings		R
	Alone	With aux. battery	

23. Measurement of Current by a Voltmeter and Shunt.—When a current I flows through a resistance R, the fall of potential over R is $E = IR$, which is in accordance with Ohm's law and our definition of the term fall of potential. In Article 15 both E and I were measured and the value of R was then computed. When R is known the experiment can be reversed and by measuring E the value of I can be computed. Thus a voltmeter may be used to measure currents in place of an ammeter. The arrangement is shown in Fig. 11.

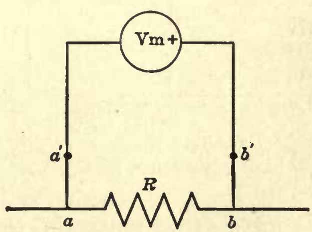


FIG. 11.—Measurement of current.

Usually the resistance which would be suitable for this purpose would be an ohm or less. Therefore any variation in the position of the voltmeter connections at a and b would cause considerable change in the resistance between these points. It is best to have these connections soldered fast, and let the voltmeter connections be made at the auxiliary points

a' and b' where a little resistance, more or less, in the voltmeter circuit will be inappreciable.

Such shunts are often made having a resistance of 0.1, 0.01, 0.001 or less, of an ohm. The current is then 10, 100, 1000, or more, times the voltmeter reading. This principle is also used in the construction of ammeters for measuring large currents. The greater part of the current is carried by a shunt of low resistance, while the delicate moving coil carries only a small current and thus in reality acts as a sensitive voltmeter. The numbers on the scale, however, instead of reading volts, are made to give the corresponding values of the currents passing through the instrument.

24. Measurement of a High Resistance by a Voltmeter Alone.—This method is a modification of the ammeter and voltmeter method for the measurement of moderate resistances, and it is based on the fact that a voltmeter is really a very sensitive ammeter. It can be used as an ammeter whenever the high resistance in series with the moving coil will be only a small part of the total resistance in the circuit.

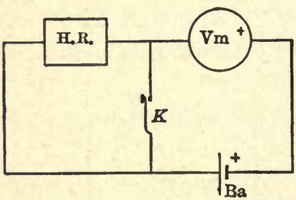


FIG. 12.—To measure the high resistance, *H. R.*

The voltmeter is joined in series with a battery and the high resistance to be measured. In this way it serves as an ammeter to measure the current through the circuit. The reading, V, of the voltmeter gives the fall of potential over its own resistance S, or in symbols,

$$V = SI.$$

Considering the entire circuit, the value of the current is given by the expression

$$I = \frac{E}{R + S}.$$

Eliminating I from these two equations and solving for R gives,

$$R = S\frac{E - V}{V}$$

The value of E, the E.M.F. of the battery, is easily measured by the same voltmeter by connecting it directly to the battery. A key arranged to short circuit the high resistance will readily change the voltmeter from its position as an ammeter to that of a voltmeter.

Record data as follows:

To Measure the Resistance of........

Vm. Zero = Vm. Resist. = ohms

Name of object measured	V	E	R

25. Time Test of a Cell.—The time test of a cell is designed to show how well the cell can maintain a current, how effective is the action of the depolarizer, and the rapidity and extent of the recovery of the cell when the current ceases. Such a test usually continues for an hour, and the results may best be shown graphically by means of curves. These curves should show the value of the E.M.F. of the cell at each instant during the test; the available E.M.F.; the current; and the internal resistance of the cell. After maintaining the current for an hour, the circuit is opened and the cell allowed to recover. The curves should show the rate and extent of this recovery.

Of course it is evident that such a continued test may prove rather severe for cells intended for only a few minutes' use at one time. And on the other hand, cells which make a good showing under the test might not be the best for long continued service. Nevertheless such a test gives the most informa-

tion regarding the behavior of the cell that can be obtained in the short space of 2 hours.

The set up for making a time test is the same as that for measuring the internal resistance of the cell, save that the battery circuit remains closed all of the time except when it is opened for an instant to measure the E.M.F. of the cell. With the proper preparation beforehand it is not difficult to observe all the necessary data and record it in a neat and convenient form.

Readings should be taken once a minute. Practice in doing this should be obtained by measuring the E.M.F. of the cell before any current is drawn from it. When ready to commence the test proper, the circuit is closed through 5 or 10 ohms, or about as much resistance as the internal resistance of the cell, and as soon thereafter as possible the first reading for the available E.M.F. is taken. One minute later the circuit is opened just long enough to take a reading of the E.M.F. of the cell. Thus every 2 minutes the available E.M.F. is recorded and on the intermediate minutes the value of the total E.M.F. is measured.

From this data two curves are plotted—one showing the variation in the E.M.F. of the cell during the test, and the other showing the same with respect to the available E.M.F. The internal resistance is computed at 5-minute intervals, the values of E and E' being taken from the curves. The current is computed from the values of R and the available E.M.F.

After the first hour of the test the battery key is changed so as to keep the circuit open, except for an instant each minute when it is closed long enough to read the voltmeter. The keys can be worked by hand and if proper care is exercised good results may be expected. Better results may be obtained by using a special battery testing key, or a pendulum apparatus which will close and open the keys in precisely the same manner each time.

During the second hour the battery will recover from the

effects of polarization, more or less completely, and at the end of the 2-hour test the value of E should be about the same as at the beginning. The recovery curve may be plotted backward across the sheet containing the other curves, thus showing very clearly the extent of the recovery.

The data may be recorded as below.

TIME TEST OF A........CELL

Time of day hour minute	E	E'	I	r

CHAPTER II

BALLISTIC GALVANOMETER AND CONDENSER METHODS

26. Capacity.—Elementary Ideas.—A given quantity of air will fill a certain vessel to a definite pressure, this pressure depending upon the amount of air and the capacity of the vessel. The more the air and the less the capacity of the vessel, the higher the pressure to which the former will be subjected.

In electricity, when a conductor is charged with a quantity Q, of electricity, it is raised to an electrical pressure, or potential, V. This potential will be greater or less according to a property of the conductor which is called its electrical capacity. The relation between Q and V is expressed by the formula,

$$Q = C\,V,$$

where C denotes the capacity measured in certain units. Since the units of Q and V have been previously established this relation shows what value of the capacity should be taken as unity. A conductor has unit capacity, therefore, when unit quantity of electricity raises it to unit potential.

Definition.—By the capacity of a conductor is meant its ability to hold a quantity of electricity. It is measured by the number of coulombs per volt required to charge the conductor.

When a body is charged with a quantity $+Q$, there is a complementary quantity, $-Q$, on the surrounding surfaces. If these surroundings are not far off the effect of this charge, $-Q$, will be to make the potential of the conductor less than it otherwise would be. In other words, the capacity of the conductor has been increased by the presence of the other conductors near it. It will now require a larger charge to raise it to the same potential as before. As more water can be

contained in a given vessel by condensing it from a vapor to a liquid, so, by analogy, this arrangement of conductors whereby the charge is increased without increasing the potential is called a *condenser*. This name does not mean that electricity is condensed, like steam into water. The analogy is only an apparent one and must not be pushed too far.

27. Condensers.—In order to make the capacity as large as possible condensers are constructed with broad sheets of tin foil placed as near as possible to other similar sheets. Actual contact is prevented by thin layers of mica, glass or paraffined paper. Large capacities are formed by building up alternate sheets of tin foil and dielectric, every other sheet of tin foil being connected to one terminal post, and the intermediate ones to the other terminal (Fig. 13).

Fig. 13.—The tin foil plates of a condenser.

The best condensers and those intended for standards are made with thin sheets of mica as insulation between the sheets of tin foil. Very good condensers are made with paper insulation, the whole pressed firmly together and boiled in paraffine until all the air and moisture has been expelled, when the whole is allowed to solidify.

A charge "Q" in a condenser means that there is a charge of $+ Q$ coulombs on one set of plates, and an equal, though negative, charge of $- Q$ coulombs on the other set of plates. We can think of the discharge of the condenser as consisting of the $+ Q$ going over to the other side and neutralizing the negative charge, thus giving rise to a positive current through the discharging circuit; or, if the negative charge passed through the circuit in the other direction, the effect would still be that of a positive current; or, if we prefer to think in terms of a "two fluid" theory and thus consider the positive charge as coming out from one side of the condenser and meeting the negative charge as it comes out of the other side, the two neutralizing somewhere in the conductor, the effect is still the

same. This illustrates how the same set of observed facts can be explained equally well in terms of widely different theories. If either charge is "real," it is probably the $-Q$, and the negative electrons travel in the opposite direction to the "current."

28. Unit Capacity.—In the international units the "unit of capacity is the *international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.*"

The farad is thus 1,000,000,000 times smaller than the C.G.S. unit of capacity. But even then the farad is far too large for ordinary use, and it is customary to express capacities in terms of a smaller unit, the *microfarad*, which, as its name indicates, is one millionth of a farad.

Problem.—A 3-microfarad paraffined paper condenser is about one foot square and an inch in thickness. How large a pile of such condensers would have a capacity of one farad?

How large a pile would it take to have a capacity of one C.G.S. electromagnetic unit?

Note.—The capacity of a solid metal sphere of 1 cm. radius is one C.G.S. electrostatic unit, when it is far from all other conductors.

29. Ballistic Galvanometer.—A ballistic galvanometer is one in which the moving system, whether coil or magnet, is made comparatively heavy and massive so that it will swing slowly. Such a galvanometer is designed to measure, not steady currents like an ammeter, but transient currents which may exist for only a very small fraction of a second. Indeed the duration is so short that it is customary not to speak of them at all as currents, but only to consider the total quantity of electricity that has passed.

Thus there are two reasons for having a slow moving galvanometer. In the first place, it is most sensitive when in the position of rest and therefore should not turn appreciably from this position before all the electricity has been able to flow through the galvanometer and exert its full effect in turning the

coil. In the second place, the coil gives one kick and then settles back to the position of rest again and the only thing that can be measured is the maximum deflection which it attains. Therefore it must move slow enough to enable one to read the deflection at the end of its swing.

For small deflections the maximum swing is proportional to the quantity of electricity, and the proportionality factor, or the quantity of electricity per millimeter of deflection, is called the constant of the galvanometer. It is determined by discharging through the galvanometer a known quantity and noting the resulting deflection. Then,

$$Q = kd,$$

where k is the desired constant. Knowing k for a galvanometer, any other quantity can be measured by sending it through the galvanometer and noting the corresponding deflection.

30. Use of Ballistic Galvanometer and Condenser.—When the poles of a battery are joined to the plates of a condenser the latter becomes charged, as explained above. The amount of this charge depends upon the electromotive force, E, of the battery and the capacity, C, of the condenser being given by the relation

$$Q = C\,E$$

When the condenser is discharged through a ballistic galvanometer it will produce a deflection d, proportional to Q or

$$Q = kd,$$

where k is the constant of the galvanometer.

If this operation has been carefully arranged so that there has been no leakage or discharge of electricity elsewhere, it is evident that the quantity that has been discharged through the galvanometer is the same quantity that was put into the condenser by the battery. That is,

$$CE = kd$$

This is a very useful relation, since it can be used to compare the value of any one of the factors involved when the other three are known or can be measured.

The best arrangement for using a ballistic galvanometer with a condenser is shown in the figure, where G represents the galvanometer, C the condenser, with the battery at B. These are connected through the key as shown. *Note that the tongue of the key is connected to the condenser, and to the condenser only.* This precaution is necessary in order that by no possibility can the battery ever be joined directly to the galvanometer. Arranged as shown, when the key is depressed the condenser is joined to the battery and becomes charged. When the key is raised the condenser is joined to the galvanometer and the charge passes through it producing a deflection.

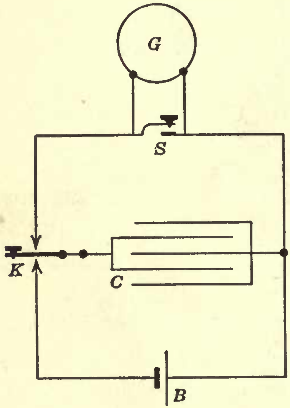


Fig. 14.—Use of a condenser.

31. Damping of a Galvanometer. Critical Damping.—After the galvanometer has given its deflection the very fact that the moving system is massive, and at the same time can move freely, which is essential for a good ballistic galvanometer, makes it very slow in coming to rest again. It will swing back and forth many times until its energy has been used up in friction against the air, and in other ways, when it will finally settle down at rest. That the swings decrease at all is due to the *damping* of the motion, as this effect of friction, etc., is called.

If the damping is increased, as would be the case if the coil were surrounded with oil, the swings would decrease more rapidly and the coil would quickly come to rest. Such damping might be so great that there would be no swings, and the coil would slowly creep back to the position of rest,

possibly taking longer to do so than when it is allowed to swing freely.

Thus it is seen that there is some intermediate value of the damping that would allow the coil to swing back to rest not too slowly and yet bring it to rest without its swinging to the other side. This value of the damping is called *critical damping*, and with this damping the coil is brought to rest in the minimum time.

The most convenient way to increase the damping of a ballistic galvanometer is to join its terminals by a key or a low resistance, as shown at S, Fig. 14. Due to the motion of the coil in a strong magnetic field an induced current will flow through S, when closed, and the supply of energy in the coil is quickly dissipated as heat by the electric current in the wire. Often S is a resistance adjusted to such a value as to give critical damping. The galvanometer can then be used once a minute, or oftener if desired.

The effect of the shunt in reducing the deflections is discussed in the next chapter.

32. The Constant of a Ballistic Galvanometer.—For the complete theory of the ballistic galvanometer and a full discussion of the various factors entering into k, the reader is referred to the chapter on the measurement of capacity. It is shown in Article 118 that

$$Q = \frac{TF}{2\pi}\left(\frac{S'}{S''}\right)^{\frac{1}{2}} d = kd.$$

Suffice it here to observe that k is a constant, resulting from the combination of the various constants in this expression; and without knowing the value of any separate factor, the value of k can be readily determined as shown below.

The galvanometer, condenser, and battery, are connected as shown in Fig. 14. The scale and telescope should be adjusted so that both the divisions and the numbers on the scale are distinctly seen in the telescope. The eyepiece must be focused on the cross hair of the telescope, which should appear very

clearly defined. When focusing the telescope upon the image of the scale as seen reflected from the mirror of the galvanometer, it must be remembered that the image is not on the surface of the mirror, but lies as far back of the mirror as the scale is in front of it.

After the setup has been tried and found to work correctly with an old dry cell for B, a standard cell whose E.M.F. is much more exactly known may be substituted for it. Ten independent determinations of d should be made, and the mean value used in the computation for k. If the scale is movable it will be necessary to measure the distance of the scale from the mirror of the galvanometer and compute the value of d for a standard distance of 1 meter.

The relation given in Article 30 is

$$CE = kd.$$

In the present case the condenser used must be one of known capacity. Likewise E must be known as exactly as the value of k is desired. And d is measured. Hence the computed value of k is,

$$k = \frac{CE}{d}$$

If the value of C is given in microfarads, k will be expressed in microcoulombs per scale division. A microcoulomb is one millionth of a coulomb and is the quantity of electricity that is represented by a current of one ampere flowing for one millionth of a second. The data can be recorded as follows:

Galvanometer			Deflection for scale at 1 meter	C	E	k
Zero	Reading	Deflection				

33. Comparison of E.M.F's. by Condenser Method.—The arrangement of a condenser with a ballistic galvanometer, may be conveniently used to measure the E.M.F. of a battery, or any other difference of potential. It thus serves as a voltmeter, and has the advantages over the ordinary voltmeter in that it measures the total E.M.F. of the battery, no matter what the internal resistance of the latter may be.

The setup is arranged as shown in Fig. 15. When the key is depressed the condenser is charged, and by raising the key it is discharged through the galvanometer. It is immaterial whether the key works this way or whether the condenser is charged when the key is up and discharged by depressing the

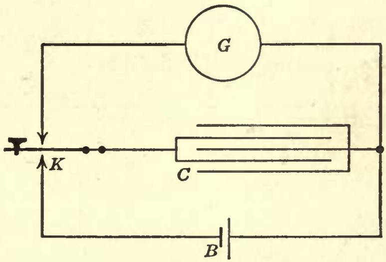


FIG. 15.—To measure the E.M.F. of B.

key. It is absolutely necessary, however, that the tongue of the key be joined to the condenser as shown in Fig. 15.

The relationship given above is $CE = kd$. Solving this for E gives,

$$E = \frac{k}{C}d.$$

The E.M.F. of the battery is thus measured by the first throw of the galvanometer needle, which is read by a telescope and scale. The best way to determine the factor k/C is to use one cell of known E.M.F. and observe the corresponding deflection. Then,

$$E' = \frac{k}{C}d', \qquad \text{or} \qquad \frac{k}{C} = \frac{E'}{d'},$$

so that finally

$$E = \frac{E'}{d'}d.$$

Having determined this coefficient of d once for all, the E.M.F. of any cell can be measured quickly and easily by observing the corresponding deflection of the galvanometer.

Inasmuch as the reading must be caught quickly at the end of the swing it will be best to take five trials and use the mean deflection for computing the value of E. The data may be recorded as follows:

Name of cell	Galvanometer			Mean deflection	$\frac{k}{C}$	E.M.F. of Cell
	Zero	Reading	Deflection			

34. Comparison of Capacities by Direct Deflection.—The same arrangement described above and shown in Fig. 15, can be used equally well for the measurement of the capacity of a condenser. It is only necessary to go through the experiment as before, and observe the galvanometer deflection for the relation,

$$CE = kd.$$

Now replacing the condenser by another one, but using the same battery and everything else the same as before, the relation becomes,

$$C'E = kd'$$

where d' is the galvanometer deflection when the condenser of capacity C' is used. Dividing the second equation by the first gives,

$$C' = C\frac{d'}{d}.$$

If C is a known capacity then the value of C' can be determined as exactly as the flings d and d' can be measured. Each of these should be taken several times, and the mean values used in the computation.

35. Internal Resistance of a Battery by the Condenser Method.—The condenser method offers a convenient and elegant means for determining the internal resistance of a cell, the principal advantage being that polarization can be almost entirely avoided. In the ammeter-voltmeter method a considerable current must oftentimes be drawn from the cell and for a period long enough to read both instruments. Such readings seldom can be repeated, for owing to polarization the cell does not return to its original condition.

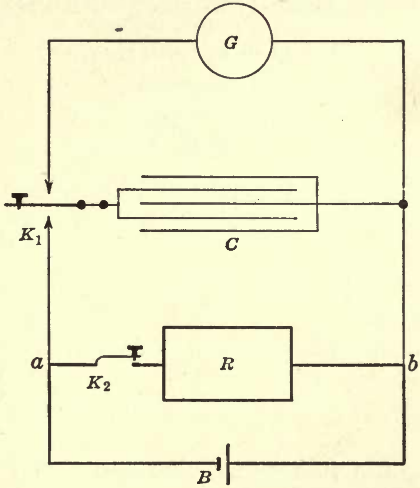


FIG. 16.—To measure the resistance of B.

The setup for using the condenser method is shown in the figure. When K_2 is closed a current flows through R and the cell, the value of which is expressed as

$$I = \frac{E}{R + r}$$

from which

$$r = R\frac{E - E'}{E'} \qquad \text{(A)}$$

where E' is written for RI, the external fall of potential. If R is known it only remains to measure E and E'.

When K_1 is depressed the condenser is charged to the E.M.F. of the cell, the quantity of electricity in the condenser being $Q = CE$. On raising K_1 the condenser is discharged through the galvanometer which gives a throw of the needle that is proportional to the quantity discharged, the relation being,

$$Q = kd = CE$$

or,

$$E = \frac{k}{C}d.$$

Similarly, if K_1 is worked while K_2 is closed the potential difference between the points a and b is RI or E', and we have,

$$E' = \frac{k}{C}d'.$$

Substituting these expressions in (A) gives,

$$r = R\frac{d - d'}{d'}.$$

It is best to use values of R that will give d' about half as large as d.

In this way it is only necessary to keep K_2 closed long enough to depress and raise K_1. With skill this interval can be reduced to less than a second when the keys are worked by hand. It is much better, however, to use a special battery testing key. This key is really several keys on one base and so arranged that a single pressure works them all in the proper order. In the present case, first K_2 would be closed, then K_1 depressed and raised, and finally K_2 opened all by a single downward pressure. This key is a regular, three-tongued, successive contact key such as is usually employed with a

Wheatstone's bridge, modified by the addition of a break contact on the top of each tongue. (See also Fig. 66 for a side view of this key.) By this means it is also a successive break key, the breaks being alternated with the makes. Thus almost any desired combination of makes and breaks can be readily obtained.

The arrangement with this key is shown in the figure, where the outline of the key is drawn, and the connections indicated for making the changes noted above. With this key any reading can be repeated as often as desired, since the cell does not become polarized in the very short time the current is flowing. By holding down the lower tongue until the rest of the key has been raised there will be no current taken from the cell as the key is being released.

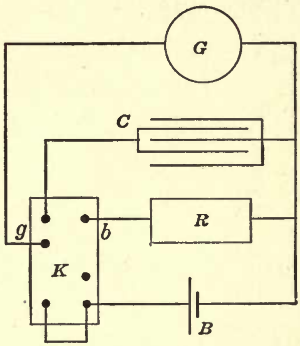


FIG. 17.—Use of special testing key.

Depressing the whole key, then, gives the deflection d'. Working the lower tongue, only, gives d.

Measure in this way the internal resistance of several cells, making five or more determinations of each cell.

The data may be recorded as follows:

Name of cell	Galvanometer				R	r
	Zero	Reading	Deflection			
			d	d'		

36. Insulation Resistance by Leakage. *First Case.*—This method is used when the resistance to be measured is so large that the current which it is possible to pass through it is too small to be measured by a sensitive galvanometer. The method consists, in brief, in letting the current flow into a condenser for a sufficient time, and then discharging the accumulated quantity through a ballistic galvanometer.

The setup is arranged as shown in Fig. 18, where R denotes the large resistance to be measured. A battery of sufficient E.M.F., E, supplies the current which flows through R and gradually charges the condenser C. When a sufficient charge is accumulated it is discharged through the galvanometer by closing the key K. The other key K', is for damping the swings of the galvanometer and bringing it to rest: and it should be kept closed all the time that the galvanometer is not being observed through the telescope.

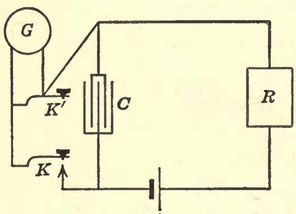


Fig. 18.—To measure a resistance of many megohms.

At any instant during the time the current is flowing the fall of potential over R is

$$RI = E - V \tag{1}$$

where V is the difference of potential across the condenser.

In this method it is assumed that the condenser has considerable capacity, and that the charging is discontinued before V has reached an appreciable part of E. If the experiment is not worked in this way the following discussion does not apply.

At the start, and as long as V can be neglected in comparison with E, the current through R is, from (1),

$$I = \frac{E}{R} \tag{2}$$

If this current flows into the condenser for t seconds, the accumulated charge is,

$$Q = It, \tag{3}$$

and when the condenser is discharged through the galvanometer there is a deflection, or fling, of d scale parts, such that

$$Q = kd \tag{4}$$

Thus the current is,

$$I = \frac{kd}{t}$$

and from (2),

$$R = \frac{E}{I} = \frac{tE}{kd} = \frac{E}{E'}\,\frac{t}{C'}\,\frac{d'}{d}$$

The "constant," k, of the galvanometer can be determined by the method described in Article 32.

If the same battery is used in finding the constant as in the experiment proper, $E = E'$, and the absolute value of the E.M.F. employed does not enter into the computation.

Then $$R = \frac{t}{C'}\,\frac{d'}{d}$$

37. Insulation Resistance by Leakage. *Second Case.*—If the high resistance has also considerable capacity it will not be necessary to use a separate condenser. Thus if it is required to measure the insulation of a condenser or a long cable the arrangement will be as shown in Fig. 19, where CR represents the condenser of capacity C and resistance R. Upon closing K, with K' also closed, the condenser is charged to the fnll potential difference of the battery. When the key is opened the charge, Q, leaks through the high resistance R. At the start, and before the charge in the condenser has been

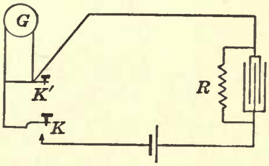


Fig. 19.—To meaure the resistance of a condenser.

appreciably reduced by the leakage, the current through R is,

$$I = \frac{E}{R}.$$

This current will reduce the original charge in the condenser by the amount

$$q = It$$

in t seconds, where t is not too great to consider the current constant during this interval. This loss of charge can be determined by recharging the condenser through the galvanometer to the original difference of potential. Then as before,

$$q = kd, \text{ and } I = \frac{kd}{t}$$

from which

$$R = \frac{E}{I} = \frac{Et}{kd} = \frac{t}{C'}\frac{d'}{d}$$

the same as before if the same battey is used to determine k.

At the beginning of this test the values of R will usually be too low because of the effect of "absorption" by which a part of the charge disappears. This reduces the charge in the condenser the same as though it had leaked out. The true value of the resistance will be obtained only after several hours, in some cases several days, but if a first test is being made it is well to determine the value of R at intervals of a few minutes. A curve plotted with the time of day for abscissæ and the corresponding values of R for ordinates, will show this variation and indicate the maximum value of the insulation resistance.

A resistance not having any capacity can be measured by this method by adding a condenser in parallel with it. But in such a case the arrangement shown in Fig. 18, would be preferable.

CHAPTER III

THE CURRENT GALVANOMETER

38. Description of a Galvanometer.—A galvanometer is a delicate and sensitive instrument for the measurement of small electric currents. All galvanometers consist of two essential parts—a coil of wire through which can flow the current to be measured, and a permanent steel magnet. In some galvanometers the coil is comparatively large and is rigidly fixed to the frame of the instrument, while the magnet is a small piece of steel suspended lightly by a fiber of untwisted silk or of quartz. In other galvanometers the arrangement is reversed. The coil is made as light as possible and is suspended by a thin strip of phosphor bronze between the poles of a large and strong magnet which often forms the body of the instrument. In either style the movable portion is made to turn as easily as possible, the amount of turning being measured by the mirror, scale and telescope method.

There are two ways of using a galvanometer. A transient current, like the discharge of a condenser, will produce a fling or kick of the galvanometer after which it will settle back to the original position. Evidently the only thing that can be measured in this case is the maximum fling. But if the current is steady the galvanometer will settle down at a deflected position, and the deflection, as the distance of this position on the scale from the position of rest is called, measures the current.

Most galvanometers are so constructed that, for small angles at least, the deflection is directly proportional to the current. That is, $I = Fd$.

The proportionality factor, F, is called the "figure of merit" of the galvanometer, and it is defined as the current per scale

division (1 mm.) that will deflect the galvanometer. The figure of merit of most galvanometers is smaller than one hundred-millionth of an ampere per millimeter.

Inasmuch as the deflection will vary with the distance of the scale from the mirror, this distance should be made 1 meter. If for any reason the scale is at a different distance the observed deflection must be corrected to what it would have been had the scale been at the proper distance.

39. Figure of Merit. (*a.*) *By Direct Deflection.*—In order to determine the figure of merit it is necessary to send a small known current through the galvanometer and observe the steady deflection it produces. The method can be understood by reference to Fig. 20. The galvanometer is joined in series with a battery, a large resistance and a key. When the key is closed, the current flowing through the circuit, and therefore through the galvanometer, is, by Ohm's law,

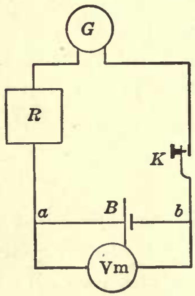


Fig. 20—To determine the figure of merit of G.

$$i = \frac{E}{g + b + R}$$

where E denotes the E.M.F. of the battery, and g, b, R, the resistances of the galvanometer, battery, and R, respectively. If this current produces a steady deflection of d scale divisions (mm.), the figure of merit is $F = i/d$.

By using a voltmeter to measure directly the fall of potential between a and b, the current i will be given by

$$i = \frac{V}{R + g}$$

where V is the voltmeter reading. Then

$$F = \frac{V}{(R + g)\, d}.$$

Thus by the use of the voltmeter, the somewhat uncertain E.M.F. of the cell is replaced by the definite voltmeter reading,

and the unknown resistance of the cell does not appear in the equations. Of course V and d must be simultaneous values, and it is understood that d is the steady deflection produced by the steady current i.

40. Figure of Merit. (*b*) *By Fall of Potential.*—With a sensitive galvanometer it is usually not possible to make R large enough to use the simple method. It is then most convenient to use a value for E which is only a small fraction of the E.M.F. of the cell. This can be done by the fall of potential method shown in Fig. 21. The fall of potential between a and b is now PI instead of E, and both P and I can be made as small as necessary. For most galvanometers it is convenient to make $P + Q = 1000$ ohms, and R about 100,000 ohms.

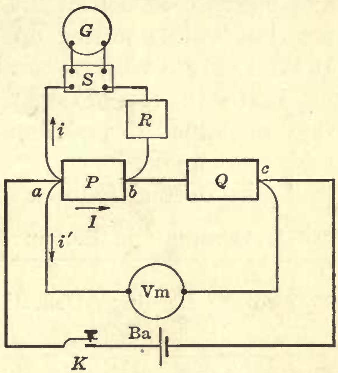


Fig. 21.—Figure of merit by fall of potential method.

The current from the battery divides at a, one part, i, going through the galvanometer, another part, I, through P, and the third part flows through the voltmeter. The fall of potential from a to b across P is the same as through R and the galvanometer, or

$$PI = R'i \tag{1}$$

where R' is the combined resistance of R and the galvanometer with its shunt, if one is used.

The current through Q is $I + i$, and the fall of potential from a to c, which is measured by the voltmeter is

$$PI + Q(I + i) = V \tag{2}$$

Eliminating I between (1) and (2), and solving for i,

$$i = \frac{P\ V}{QR' + PR' + PQ}$$

Then as before,

$$F = \frac{PV}{(P+Q)\,R' + PQ} \cdot \frac{1}{d}$$

Usually the term PQ can be omitted in comparison with QR', since P is much less than R', and by keeping P small the error thus introduced may be made less than the error in reading the voltmeter or the galvanometer deflection. The use of the shunt S, is to protect the galvanometer from large and unexpected deflections. It should be kept on the "zero position" while setting up or taking down the connections. It is always a wise precaution not to connect in the battery until after the rest of the setup is completed and has been carefully examined to make sure that no unintended connections have been made.

Record data as follows:

To Determine the Figure of Merit of Galvanometer No....

Zero	Reading	Deflection	R	V	P	Q	F

41. Other Constants of the Galvanometer. *Megohm Sensibility.*—The sensitiveness of a galvanometer is often expressed in megohms (millions of ohms). This means the number of megohms that can be placed in series with the galvanometer to give a deflection of one scale division per volt of the applied E.M.F. Expressed in ohms, its numerical value is evidently given by the reciprocal of the figure of merit, so when the latter is given the megohm sensibility is readily computed.

Voltage Sensibility.—For some purposes it is necessary to know the sensitiveness of a galvanometer to differences of potential applied to its terminals. This will depend largely upon the resistance of the galvanometer, as a small resistance will allow a larger current to flow. The voltage sen-

sibility is expressed in volts per scale division, and is given by the product of the figure of merit and the resistance of the galvanometer.

42. Use of Shunts. *Common Form.*—When the current to be measured by any instrument is larger than the range of the scale the latter can be increased to almost any desired extent by placing a shunt in parallel with the instrument, as shown in Fig. 22. The shunt and instrument thus form two branches of a divided circuit and the current through one branch is directly measured. Knowing the current in this branch the total current can be computed.

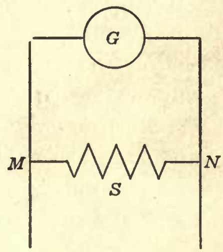


FIG. 22.—Use of a shunt.

Thus let G denote the galvanometer or ammeter, and S the shunt. The current through the galvanometer is, by Ohm's law,

$$i = \frac{V}{g}$$

where V is the potential difference between M and N, and g denotes the resistance of the galvanometer. In the same way the total current, which flows through g and s in parallel, is

$$I = \frac{V}{\frac{gs}{g+s}} = i\frac{g+s}{s}$$

The ratio, I/i, of the main current to that part which is measured by the galvanometer is called the "multiplying power of the shunt." From the relation above it is seen to be equal to $\frac{g+s}{s}$. It is the factor by which the current measured by the galvanometer must be multiplied to give the total current through the main circuit. In order that this factor may be expressed in convenient round numbers, 10, 100, 1000, etc., it is necessary to have a series of shunts care-

fully adjusted to $\frac{1}{9}, \frac{1}{99}, \frac{1}{999}$, etc., of the resistance of the galvanometer. Such shunts will not have the same multiplying power when used with a galvanometer of different resistance, and therefore can be used advantageously only with the galvanometer for which they were made. Placing a shunt around a galvanometer will reduce the total resistance of the circuit and therefore the current measured by the galvanometer times the multiplying power of the shunt does not give the value of the original current, but the value of the new main current. Sometimes extra resistances of $0.9g$, $0.99g$, and $0.999g$, are inserted to keep the total resistance of the circuit constant.

When the galvanometer is used ballistically these shunt ratios are not the same as for steady currents because of the varying amounts of damping produced.

43. Universal Shunt.—Another way of using a shunt is to connect a resistance as a permanent shunt across the terminals of the galvanometer. The current to be measured is passed through only a part of this shunt the remainder acting merely as resistance in series with the galvanometer. Fig. 23 shows a four-step shunt, the total resistance of all being S, which is joined as a permanent shunt on the galvanometer. The multiplying power of this arrangement is

$$\frac{(g + S - s) + s}{s} = \frac{g + S}{s}$$

where S is always the same constant resistance. Therefore in changing from one shunt to another the numerator remains constant and the change in multiplying power depends only upon the changes made in s. Furthermore, no error is introduced by contact resistance at b as any resistance at this place does not affect the accuracy of the shunt ratios. The total resistance of the shunt should be about twenty times that of the galvanometer.

A universal shunt box consists of several coils permanently

joined in series. When used as a shunt the galvanometer terminals are connected to the extremities of this series, thus using the entire resistance. The current is not passed through all of the resistance in the shunt box as in the case of a common shunt, but it may be passed through either one or several of the coils constituting the series. Thus the part which actually carries the current is the real shunt, while the remaining coils are thrown in with the galvanometer to further aid in

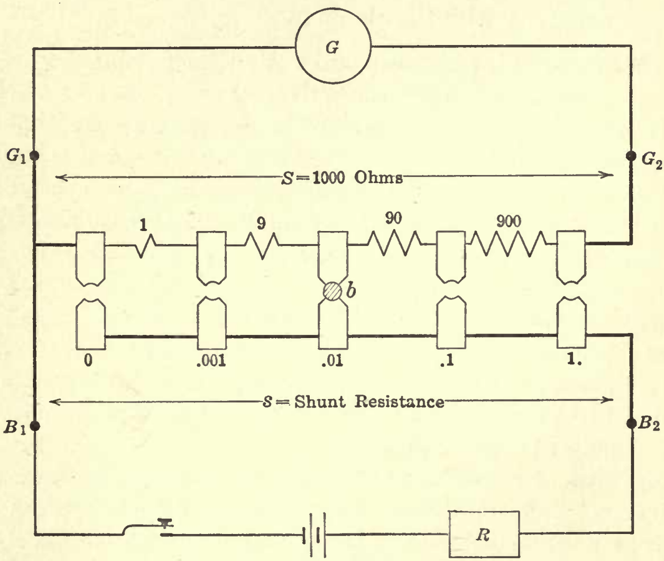


FIG. 23.—Diagram illustrating the universal shunt.

reducing the deflection. The arrangement will be clearly understood by referring to the figure, which shows the connections of a universal shunt having a total resistance of 1000 ohms. The figure shows the shunt set for a multiplying power of 100. G_1, G_2, B_1, B_2, are binding posts where connections to the external circuits are made.

The change produced in the resistance of the total circuit is not as easily determined as for a common shunt. Indeed

the resistance is frequently greater with the shunt than when the galvanometer is used alone. In many kinds of work it is not essential that the resistance shall be constant or even known. Where it must be known it can be determined for the galvanometer and its shunt combined as readily as for the galvanometer alone.

Shunts are marked with the numbers 0.1, 0.01, 0.001, implying the fractions of the current which they pass through the galvanometer, or with the numbers $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$ implying the ratio between their resistance and that of the galvanometer.

It is evident that when the universal shunt is used at the point marked 1. the galvanometer is not quite as sensitive as with no shunt connected. If S is several times g this slight reduction in the sensitiveness is of small moment. The essential thing is that when the shunt is set at 0.1, 0.01, etc., the same total current will give deflections 0.1, 0.01, etc., as large as with the shunt set at 1. And this arrangement of the galvanometer shunts is especially useful because a given series of shunts will have the same relative multiplying powers when used with any galvanometer. Since the damping is constant, the shunt ratios remain the same when the galvanometer is used ballistically.

Any ordinary resistance box having a traveling plug for making a third connection at any intermediate point can be used as a universal shunt for any galvanometer. For all values the shunt ratios are very accurate since all the coils are even ohms and can be adjusted much more precisely than in the case of common shunts. Differences in temperature between the galvanometer and shunt produce no error, but should remain constant while measurements are being made.

44. The Multiplying Power of a Shunt. *Test of a Shunt Box.*—The effect of a shunt in increasing the amount of current that can be measured by a galvanometer can be determined experimentally by finding the figure of merit of

the galvanometer alone, as shown in Article 40 and then redetermining it for the galvanometer with its shunt, considering both together as a new instrument having a resistance of $\frac{gs}{g+s}$. The ratio of these two figures of merit is the multiplying power of the shunt.

In the same manner all of the shunts in the shunt box should be tested, and the multiplying power of each one determined. The results should be compared with the values stamped on the shunt box, and also with the computed values as determined by the relative resistances of the galvanometer and the shunt.

When using the shunts of lowest resistance it may be better to determine the figure of merit by the direct deflection method, Article 39. The scale deflection should be about the same for each shunt.

45. Resistance of Galvanometer by Half Deflection. (*a*) *Resistance in Series.*—In some of the foregoing exercises it is necessary to know the resistance of the galvanometer as it has been used, either alone or combined with a shunt. If this is unknown it can be determined with a fair degree of accuracy by the method of half deflection.

Let the galvanometer be connected to a source of small potential difference, of such amount that a large deflection can be obtained with only the galvanometer resistance, G, in the galvanometer branch. Now add enough resistance, R, in series with the galvanometer to make the deflection exactly one-half of its former value. This means that the current has been reduced to half its former value and therefore the resistance, $R + G$, in the galvanometer circuit has now been made twice as much as when the galvanometer was used alone. That is,

$$R + G = 2G \qquad \text{or} \qquad G = R.$$

To avoid the errors arising from thermal currents, etc., it is best to reverse the battery and repeat the measurements, taking the mean of the two results as the correct value of G.

46. Resistance of a Galvanometer by Half Deflection. (*b*) *Resistance in parallel.*—When the resistance of the galvanometer is low, or a small E.M.F. is not readily available, the galvanometer may be placed in series with a battery of one or more cells and sufficient resistance to give a fairly large deflection. Then let a resistance box be joined in parallel with the galvanometer, and the resistance varied until the deflection is just half of its former value. The current is now divided between the galvanometer and its shunt, half of the original current flowing through the galvanometer and the rest through the shunt. If the main current is unchanged, this means that the current is equally divided between the galvanometer and its shunt. From this it follows that the resistance of the galvanometer is equal to the resistance of the shunt.

It is true that the addition of the shunt has reduced the resistance of this portion of the circuit, but as this is only a small part of the total resistance in the battery circuit, the main current in the second case, and which is divided between the galvanometer and the shunt, will be larger than the current in the first case by less than can be read on the galvanometer scale. This point can be tested by placing the shunt resistance in series with the galvanometer and noting whether it produces an appreciable effect in reducing the deflection.

Problem.—Let the student draw a setup illustrating this method. Let him give a mathematical proof that $G = S$.

47. Differential Galvanometer.—A differential galvanometer has two independent coils, as nearly alike as possible. A current passed through either one will produce a deflection, and if the current flows through both coils in series the deflection is due to the effect of both coils acting together. If the effect of the current in one coil acting alone is to produce a deflection in one direction, and the effect of a current in the other coil is to produce a deflection in the other direction, the effect of both coils acting together will be the difference of the

two, and the resulting deflection will be smaller than for either coil alone. If the coils have been carefully made, and adjusted so that the magnetic effect of each upon the needle is the same, then no deflection will be produced by equal currents flowing through the two coils in opposite directions; for the effect of one coil is just neutralized by the other. Usually this balance is not exact, and a final adjustment is required before using the galvanometer. And since it is impossible to have the coils exctly alike, the two currents will not be equal for a balance.

Calling the current through one coil I', and that through the other I'', the relation between them would be,

$$I' = nI'', \tag{1}$$

where n is a constant whose value is about unity.

To compare two resistances R and X, they are joined in parallel with each other, as shown in the figure. In series with each is one coil of the galvanometer. A resistance, P, is placed in series with the battery in order to keep the current from being excessive.

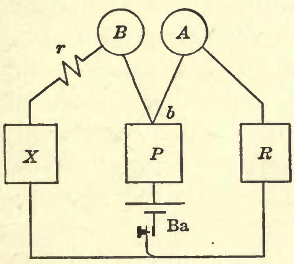


FIG. 24.—Use of the differential galvanometer, AB.

The galvanometer is adjusted as follows. R and X are short circuited by inserting all of the plugs, or otherwise. This is better than removing them from the circuit, as no change in connections will be required when they are to be used. On now closing the key the current through the galvanometer coils will depend only upon their relative resistances, and probably these are not such as will give a balance. Let a resistance, r, be now added to the side having too large a current and adjusted to give a balance. Then, from (1),

$$\frac{V}{A} = n\frac{V}{B + r} \tag{2}$$

where A and B are the resistances of the two coils, and V is the fall of potential between a and b.

Inserting R and X ,and adjusting R till a balance is again obtained, gives, from (1),

$$\frac{V'}{A+R} = n\frac{V'}{(B+r)+X} \tag{3}$$

where V′ is the new value of V.

Exchanging R with X, and readjusting R to again balance gives

$$\frac{V''}{A+X} = n\frac{V''}{(B+r)+R'} \tag{4}$$

where R' and V'' are the slightly changed values of R and V. Dividing (2) by (3) gives,

$$\frac{A+R}{A} = \frac{(B+r)+X}{(B+r)}$$

Clearing of fractions and solving for X

$$X = \frac{R\,(B+r)}{A} \tag{5}$$

Similarly from (2) and (4)

$$X = \frac{R'\,A}{B+r} \tag{6}$$

Multiplying (5) and (6) and extracting the square root

$$X = \sqrt{RR'}$$

47A. Differential Galvanometer in Shunt.—The differential galvanometer is even more useful in the comparison of two low resistances. In this case each of the galvanometer coils is used as a sensitive voltmeter to measure the fall of potential over the two resistances. If the fall of potential over each of the two resistances is the same, when they are in series, the resistances must be equal.

The galvanometer coils should be of high resistance, say

several thousand ohms. This means that they will have a great many turns of (copper) wire, which will make the galvanometer sensitive to small currents. Therefore only a little current will be shunted from the low resistances that are being compared.

For this comparison it is necessary to have a standard low resistance which can be varied by small steps. This is joined in series with the unknown resistance X, as shown in Fig. 25. Some additional resistance, P, is placed in the battery circuit to keep the current from being too large. The two coils of the galvanometer are connected as shown.

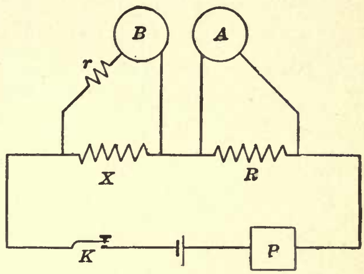


FIG. 25.—Differential galvanometer, AB, in shunt.

To adjust the galvanometer the two coils are connected together in parallel and both shunted across the same low resistance. Both coils will now have the same fall of potential and the galvanometer should give no deflection. In case there is a deflection, the current through one of the coils must be reduced by adding some resistance, r, in series with this coil until the deflection is brought to zero. This will make the resistances of the two shunts unequal, but the deflection will be zero when each shunt has the same fall of potential.

When connected as shown in Fig. 25, a balance will be obtained when R has been adjusted to equal the value of X, provided that the current through each is the same. In general this will not be the case, for by introducing r the shunt currents are not the same. Therefore a balance is obtained when the resistance of X with its shunt equals the resistance of R with its shunt. That is, when,

$$\frac{XB}{X+B} = \frac{RA}{R+A}$$

A second balance is obtained by exchanging X and R, and readjusting the value of the latter to R' for zero deflection. Then,

$$\frac{R'B}{R' + B} = \frac{X A}{X + A}$$

Dividing the first of these equations by the second gives,

$$\frac{X(R' + B)}{R'(X + B)} = \frac{R(X + A)}{X\,(R + A)}$$

or,

$$\frac{X}{R'}\,\frac{X}{R} = \frac{(X + A)\ (X + B)}{(R + A)\ (R' + B)} = 1, \text{ very nearly.}$$

Even in the extreme case of $A = 1000$ and $B = 2000$, with $X = 1$ ohm, this ratio differs from unity by only about two ten-thousandths, and the difference is correspondingly less when A and B are more nearly equal.

Therefore,

$$X = \sqrt{R R'}$$

The value of X is thus a mean proportional between the values of R required to give the two balances. If these values are nearly equal, no appreciable error is made by taking X as the arithmetical mean. In case a change of one ohm in R produces a readable deflection the tenths of ohms can be obtained by interpolation.

CHAPTER IV

THE WHEATSTONE BRIDGE

48. The Wheatstone Bridge.—The Wheatstone bridge consists, essentially, of two circuits in parallel and through which an electric current can flow. Let these circuits be represented by ABD and ACD, Fig. 26, and let the currents through the two branches be denoted by I and I'. Since the fall of potential from A to D is the same whichever path is

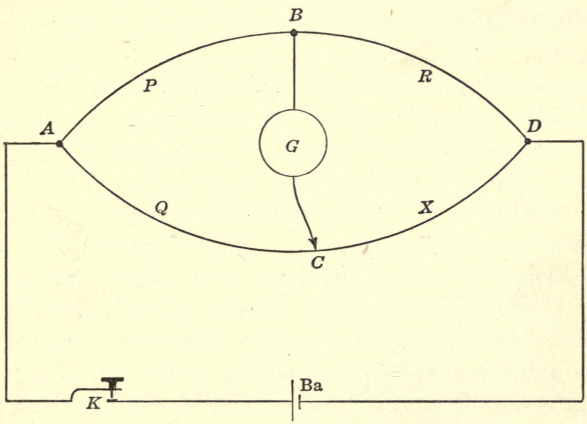


FIG. 26.—Principle of the Wheatstone Bridge.

considered, there must be a point C on one circuit which has the same potential as any chosen point B on the other. If one terminal of a galvanometer is joined to B and the other terminal is moved along ACD the galvanometer will indicate zero deflection when the point C has been found. Since B and C have the same potential, the fall of potential from A to

B is the same as from A to C, or in terms of the currents and resistances,

$$IP = I'Q$$

where P and Q are the resistances of AB and AC, respectively. Similarly for the other part of the circuits

$$IR = I'X.$$

Dividing one equation by the other eliminates the unknown currents and gives

$$\frac{P}{R} = \frac{Q}{X}$$

as the relationship of the resistances when the bridge is balanced. In the usual method of using the Wheatstone bridge three of these resistances are known and the value of the fourth is easily computed from the above relation as soon as a balance is obtained.

49. The Slide Wire Bridge—Simple Method.—The Wheatstone bridge principle is used in several forms of apparatus for the measurement of resistance. The simplest of these is the slide wire bridge as shown in Fig. 27. The unknown resistance which is to be measured is placed at X, while at R is the known resistance, usually a box of coils. The branch ACD consists of a single uniform wire, usually one meter in length, stretched alongside or over a graduated scale. The balance is obtained by moving the contact C along the wire until a point is found for which the deflection of the galvanometer is zero when K' and K are closed. This contact should not be scraped along the wire, but always raised, moved to the new point and then gently but firmly pressed into contact with the wire. Neither should it be used for a key as the continual tapping will dent the wire and destroy its uniformity. The two keys K' and K are both combined into a single successive contact key often called a Wheatstone bridge key, in which one motion of the hand will first close the battery key, and then, after the currents have been established, will close

the galvanometer circuit. The need of such a key is very evident when there is self inductance in X.

When the point C has been located we have, from Kirchhoff's second law,[1]

$$xI' = apI$$

where a is the length of the bridge wire from A to C, and p is the resistance of unit length of this wire. x denotes the value of the resistance of X. Similarly,

$$RI' = bpI$$

and dividing

$$x = R\frac{a}{b} = R\frac{a}{1000 - a}$$

if the total length of the bridge wire is 1000 millimeters.

Measure in this way the resistances of two or more coils.

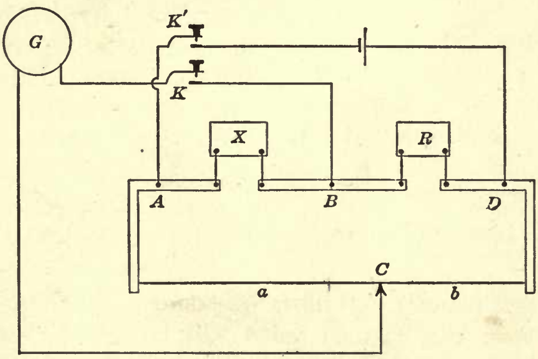


FIG. 27.—The simple slide wire bridge.

Also, measure the same coils when joined in series and compare the result with the computed value,

$$R = R' + R''.$$

When two coils are joined in parallel the measured resistance should fulfill the relation,

$$1/R = 1/R' + 1/R''.$$

[1] See page 101.

Problem 1.—Exchange the positions of the battery and the galvanometer and then deduce the formula for x, as above.

Problem 2.—Prove that for three resistances in series

$$R = R_1 + R_2 + R_3$$

and in parallel

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

Problem 3.—Deduce the corresponding expressions for five resistances.

50. Calibration of the Slide Wire Bridge.—In deducing the formula for the slide wire bridge it was assumed that the bridge wire was divided into 1000 parts of equal resistance, and that the readings obtained from the scale corresponded to these divisions. To make sure that the scale readings do thus correspond to the bridge wire it is necessary to calibrate the wire, that is, to determine experimentally what readings on the scale correspond to the 1000 equiresistance points on the wire.

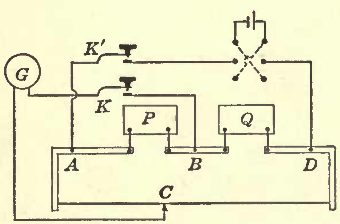


Fig. 28.—Calibration of the simple bridge.

Two well-adjusted resistance boxes are inserted in the two back openings of the bridge, and the battery and galvanometer connected in the usual manner. If now, for example, 500 ohms are put in each box the balance point will be at the middle of the bridge wire and this should be at the point marked 500 on the scale. If it is not, but falls a distance f below 500, then f is the correction which must be added to the observed reading to obtain the true reading. Since there may be thermal currents in the galvanometer circuit this value of f should be computed from the mean of two readings, one taken with the battery current direct and the other taken with the battery reversed.

In the same way the true location of the points 100, 200, 300, 400, 500, 600, 700, 800, and 900 can be found. It will be

found convenient to keep the sum of P and Q always constant at 1000 ohms.

Finally a calibration curve is drawn, with the readings of the scale for abscissæ and the corresponding corrections for ordinates. The corrections for any point on the scale can then be read directly from the curve. The corrected readings are thus expressed in thousandths of the total bridge wire, including all the resistance of straps, connections, etc., between the two points where the battery is attached.

CALIBRATION OF BRIDGE No. .....

P	Q	Scale readings			True reading	Cor.
		Battery direct	Battery reversed	Mean		

51. Double Method of Using the Slide Wire Bridge.—In the simple method above only a single balance point was obtained, and the value of the unknown resistance was computed from the relation,

$$\frac{x}{R} = \frac{a}{1000 - a}$$

where a denotes the reading on the scale at the point of balance, and is assumed to be the length of one portion of the bridge wire.

The measurement of resistance will be more precise if x and R are exchanged with each other, without, however, changing the value of either one, and a new balance point determined. This second balance point will be, say, at a' on the scale, and

$$\frac{x}{R} = \frac{1000 - a'}{a'}$$

Combining these two equations by the addition of proportions,

$$\frac{x}{R} = \frac{1000 + (a - a')}{1000 - (a - a')} = \frac{1000 + d}{1000 - d}$$

in which the actual values of a and a' do not appear, but only their difference. Thus all questions regarding the starting point of the scale or the wire are eliminated, and if d is small any error made in its determination will have only a small effect upon the value of x as computed from this equation.

52. The Wheatstone Bridge Box.—In the slide wire form of the Wheatstone bridge the balance is obtained by locating a certain point on the wire, and the accuracy of the measurement depends upon the accuracy with which the lengths of the two portions of the wire can be measured. In the Wheatstone bridge box the wire is replaced by a few accurately adjusted resistance coils. Thus while the number of ratios that can be employed is less than ten, the values of these few ratios are precise even when the ratio is far from unity. The usual arrangement is to make P and Q, Fig. 26, the two ratio arms with the unknown resistance in X and obtain the balance of the bridge by adjusting the resistance of R. The value of the unknown is then given by the usual relation,

$$X = R\frac{Q}{P}$$

and is known as accurately as are the values of P, Q and R.

In a common form of the Wheatstone bridge box, P and Q each contain 1, 10, 100, and 1000 ohm coils thus giving ratios of 1000, 100, 10, 1, 0.1, 0.01, 0.001. The rheostat arm, R, can be varied by one ohm steps from 0 to 11110 ohms. This gives a range of measurement of unknown resistances from 0.001 ohm to 11,110,000 ohms. In using such a bridge it is best to first set the ratio arms equal, say 1000 ohms each, and obtain an approximate value of the unknown resistance. Then change the ratio to such a value that R may be given in four

figures. This will give the resistance of the unknown to four significant figures also.

A more convenient form is the decade bridge. The rheostat arm is arranged on the decade plan with one plug for each decade. The resistance in this arm is indicated by the position of the plugs, which always remain in the box. The ratio arms

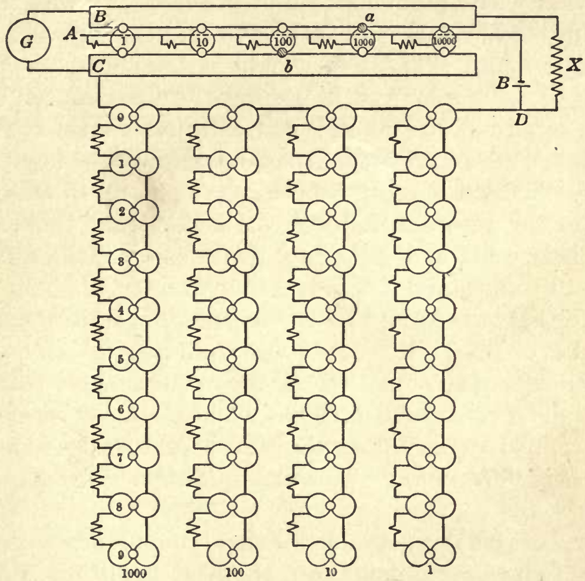


FIG. 29.—Diagram of a decade Wheatstone bridge box.

consist of a single series of coils of 1, 10, 100, 1000, 1000, 10000, ohms, but any coil can be used in either arm, (but of course the same coil can not be used in both arms at the same time). The connections not visible are clearly indicated by lines drawn on the top of the box. The different parts should be carefully compared with the diagram of Fig. 26, Article 48, and the points A, B, C, D, located before attempting to use the box. The resistance to be measured is

joined to the posts marked X, the battery and the galvanometer as shown, with a key in each circuit—preferably a successive contact key.

When starting to obtain a balance the ratio arms are each set at 1000 ohms (the figure shows only one 1000 ohm coil at a, but there are two on the box), and an approximate value of the resistance determined. This is done by shunting the galvanometer with the smallest shunt available and with R set at one ohm the keys are quickly tapped and the direction of the deflection noted. The key should not be held down long enough to cause a large deflection as the direction can be seen from a small one just as well and with less danger to the galvanometer. Next, R is set at 9000 ohms and the key tapped. Usually this deflection will be in the opposite direction. If it is not try zero and infinity. Knowing that the value of X lies between 1 and 9000, say, this range is divided by next trying 100 ohms, and if X is less than this try 10 ohms. Suppose X is between 10 and 100. This range is divided by trying 50, and so on until it is reduced to one ohm, say X is found to be between 68 and 69 ohms. Then the ratio arms are changed so as to make R come 6800 or 6900. The exact value for a balance is determined by continuing the same process and is found, say, when R is 6874 ohms. This example then gives $X = 68.74$ ohms.

If the best balance, obtained with no shunt on the galvanometer still gives some deflection, the next figure for X can be obtained by interpolation, but this is not usually required. If greater accuracy is desired it is necessary to make a second measurement with the battery current reversed through the bridge. This will reverse some of the errors, and especially the effect of thermal currents in the galvanometer branch. The mean of these two measurements will then be nearer the true value of X than either one alone.

Measure the resistance of several coils and check the results by measuring their resistance when joined in series and parallel. If some of these coils have an iron core notice the

effect of first closing the galvanometer key and then closing the battery key. Remember that the formula for this method was deduced on the assumption that all of the currents were steady, and that there was no current through the galvanometer.

The data can be recorded as follows:

Object measured	P	Q	R	X

53. Location of Faults.—By a fault on a telephone, electric light, or other line, is meant any trouble by which the insulation of the line is impaired, or which interferes with the proper working of the line. The principal kinds of faults are named as follows:

A *ground* is an electrical connection more or less completed between the line and the earth. In the case of a cable any connection from one of the wires to the lead covering of the cable constitutes a ground.

A *cross* is an electrical connection between two wires.

An *open* is a break in the line.

In testing for a fault it is first necessary to determine to which of these classes the given trouble belongs. A testing circuit is made by connecting a battery in series with a voltmeter or other current indicator. The faulty line is then picked out from among the good ones by one of the methods outlined below.

Test for Grounds.—The test to find grounded wires can be made at any point along the line. The battery side of the testing circuit just described is connected to the ground, and the wire from the voltmeter side is brought into contact successively with each wire to be examined. When the grounded wire is reached the battery circuit is completed through the earth, and this will be indicated by the deflection of the voltmeter.

Tests of this kind made with alternating current are often unreliable because of the capacity of the line.

Test for Opens.—Before testing for opens the distant ends of the wires are joined together and grounded. The test is applied at the near end, using the testing circuit in the manner just described. As each wire is tried the voltmeter will indicate the ground which has been placed on the other end, unless

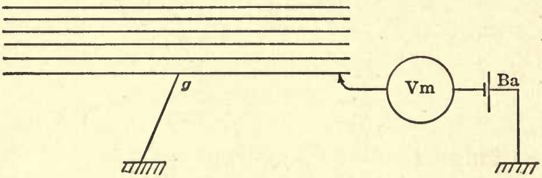


Fig. 30.—Picking out the grounded wire.

the line is open at some intermediate point. If the line is broken the needle of the voltmeter will remain at rest, showing that there is no electrical connection through that wire.

Test for Crosses.—In testing for crosses the near ends of all the wires are connected together and to one end of the testing circuit. The wires are then disconnected one at a time, and the free end joined to the other end of the testing circuit.

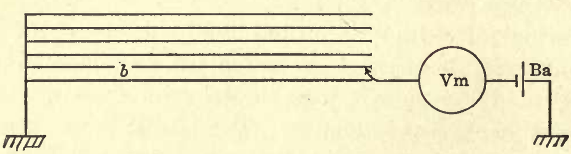


Fig. 31.—Picking out the broken line.

A movement of the voltmeter needle shows that the wire being tested is crossed with one of the other wires. If there is no indication of a cross the wire is laid at one side and the test continued with other wires until all the good wires have been removed, and only crossed wires remain.

In case there are several sets of crossed wires among those remaining, the near ends of all the crossed wires should be

separated. Then connecting them, one at a time, to the battery side of the testing circuit, touch each of the other wires in succession with the wire from the voltmeter. A deflection indicates that the line touched is crossed with the one joined to the battery.

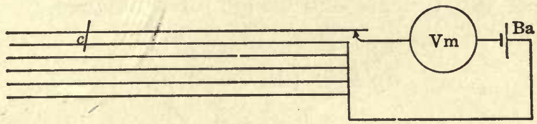


FIG. 32.—Picking out the crossed lines.

Of course the wires must not be connected at the other end, or through any switchboard as this would give the test for a cross.

54. Methods for Locating Faults.—The usual methods for locating faults in a telephone line or cable are based on the principles of the simple slide wire bridge. The following methods are illustrative examples of this kind of measurement and show the general mode of procedure. They are called loop tests because the wire being tested is joined to a good wire thus forming a long loop out on one wire and back on the other.

In case the line is open so it can not be used as one arm of a Wheatstone bridge, the position of the break can be located by comparing the capacity of the line out to the break with the capacity of similar lines whose length is known.

55. The Murray Loop.—In the Murray Loop method the grounded wire is joined to one end of a slide wire bridge, as shown at *A*, Fig. 33. A second wire of the same length and resistance and similar to the first, but free from faults, is joined at *D* to the other end of the bridge. These two wires are joined together at the far end of the line, thus forming the loop. This loop is divided into two portions by the fault at *F*, which in the figure is assumed to be a ground. These two parts form two arms of a Wheatstone bridge, the other two being formed by the bridge wire *ACD*. It is usually best to connect the galvanometer and battery in the position in-

dicated, the battery connection at F being made through the earth.

Let C indicate the balance point. Let p denote the resistance of 1 mm. of the bridge wire, and p' the resistance of unit length of the wire forming the loop. Then by the principle of the Wheatstone bridge for a balance,

$$apI = (c - a)\,pI'$$

and

$$dp'I = (2L - d)\,p'I'$$

from which

$$d = 2L\frac{a}{c}$$

where d is the distance to the fault and L is the length of the faulty wire.

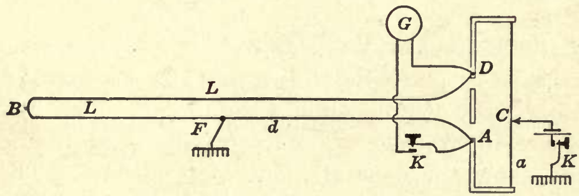


FIG. 33.—Locating the position of the ground, F.

This determination can be checked by exchanging the good and bad wires in the arms of the bridge. This change requires a slight modification in the formula used in solving for d.

56. Fisher's Method.—It sometimes happens that no good wire like the grounded one can be obtained. It is still possible to locate the fault provided only that two good wires can be obtained, the only requisite being that they terminate at the same point as the faulty wire.

First make connections as in Fig. 33 using one of the good wires to complete the loop with the faulty wire. Then as before,

$$\frac{a}{c} = \frac{dp'}{Lp' + Hp''} \qquad \text{(A)}$$

where Hp'' is the resistance of the good wire.

Next use the other good wire to connect the battery to the far end of the line as shown in Fig. 34 and obtain a second balance. The ground at F will make no difference if there is not a second ground at some other point In this case, if a' is the reading,

$$\frac{a'}{c} = \frac{Lp'}{Lp' + Hp''} \tag{B}$$

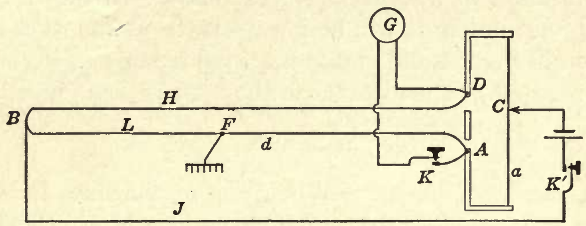


FIG. 34.—Finding the relative resistance of the two lines.

From (A) and (B),

$$\frac{a}{a'} = \frac{d}{L} \quad \text{or} \quad d = L\frac{a}{a'}$$

where as before, d is the distance to the fault and L is the length of the faulty wire.

57. Location of a Cross.—The methods for locating a cross are similar to those just given for locating a ground. The

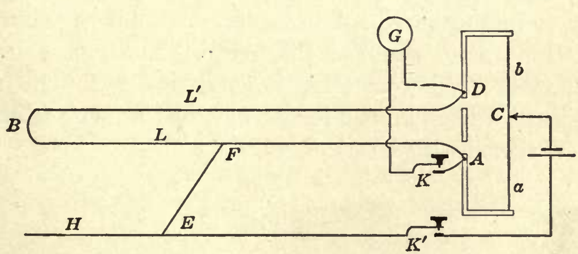


FIG. 35.—Locating the position of the cross, EF.

only difference is that instead of connecting the battery to F by means of the earth, as shown in Fig 33, the connection is

now made by means of the wire which is crossed with the line AB. The balance point is found the same as before, and the distance to the fault computed by the formulæ given above. Fig. 35 shows the Murray loop for locating the cross EF between the lines L and H. Of course the location of the cross could have been made equally well by using line H in the bridge in the place of L.

58. Location of a Cross. *Wires unlike.*—In case the lines L and L' are unlike it will be necessary to use another line J as shown in Fig. 34 and obtain a second balance as in the preceding method. The distance to the cross is then given by the formula $d = L\frac{a}{a'}$, derived as before.

59. Location of Opens.—When one of the lines is broken it will not be possible to use it for one arm of a Wheatstone

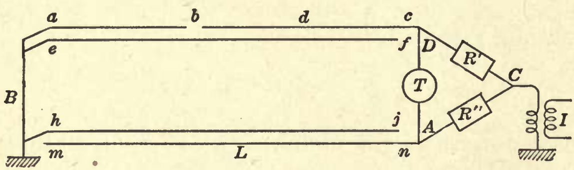


FIG. 36.—Finding the distance out to the break at b.

bridge. If the line is one of a pair it may have sufficient capacity to be measured. The simplest way is to charge one piece of the broken wire and discharge it through a ballistic galvanometer. This deflection is compared with the deflection obtained when a known length of a similar wire is charged and discharged in the same way. Then

$$d = L\frac{d'}{d''}$$

Usually a more exact determination can be made by the bridge method (Article 109) Let ac and ef represent the two wires of a pair, of which ac is broken at the point b. A similar pair is shown by mn and hj. The line mn and the part bc

of the broken line are joined to the non-inductive resistances R' and R'', as shown. The other wires of these pairs, and the remainder of the broken wire, are joined to the ground. An induction coil, I, or some other source of alternating E M.F., is used to charge the lines through the resistances. When the latter are adjusted to give a minimum sound in the telephone T,

$$d = L\frac{R''}{R'}$$

60. Resistance of Electrolytes.—When a current flows through an electrolyte it is accompanied by a decomposition

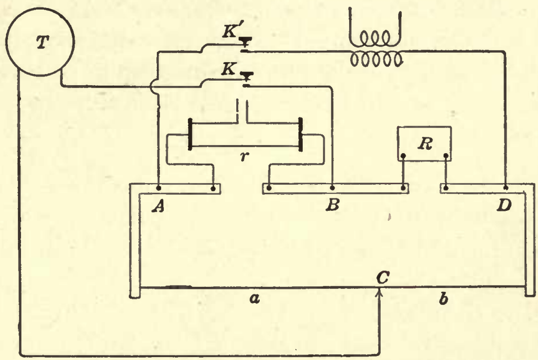


FIG. 37.—Resistance of an electrolyte.

of the substance in solution. The positive ions move in the same direction as the current, while the negative ions travel in the opposite direction, each being liberated at the electrodes. In general this action causes polarization, which tends to oppose the flow of current. In order, therefore, to measure the resistance of an electrolyte it is necessary to employ an alternating current. This can be most readily obtained from a small induction coil.

The electrolyte is placed in a suitable cell, and made the fourth arm of a Wheatstone bridge, the induction coil

being used in place of the usual battery. The resistance of the electrolyte can then be determined by the Wheatstone bridge method in the usual way, and when the bridge is balanced,

$$r = R\frac{a}{b}$$

Since an alternating current is employed, this balance can be found by means of a telephone receiver connected in the usual place for a galvanometer. For purposes of instruction the best form of cell for holding the electrolyte is a cylindrical tube with a circular electrode closing each end. The resistance measured by the bridge is then the resistance of the electrolyte between the two electrodes, and knowing the resistance of this column of the electrolyte the resistivity, s, of the solution can be calculated the same as for metallic conductors, or,

$$s = r\frac{A}{L}.$$

Where A is the cross section of the tube containing the solution and L is the distance between the electrodes.

The conductivity of the solution, c, is the reciprocal of this, or,

$$c = \frac{1}{s} = \frac{L}{rA}.$$

Since the resistance of an electrolyte, or more strictly, its conductivity, depends upon the amount of the substance in solution—that is, upon the number of ions per cubic centimeter—if we wish to compare the conductivities of different electrolytes it is necessary to express the concentrations in terms of the number of ions per cubic centimeter. This is usually stated in terms of the number of gram molecules of substance that are dissolved in one liter of the solution. For the purpose of this experiment it is necessary to express the concentrations in terms of the number of gram-molecules in 1 cc of the solution. The molecular conductivity, μ, of

an electrolyte is then defined as the conductivity per gram-molecule of salt contained in each cubic centimeter of solution.

$$\mu = \frac{c}{m} = \frac{1}{ms} = \frac{L}{mrA} = \frac{bL}{amRA}$$

where m = number of gram-molecules in 1 cc. of the solution.

The most interesting application of the conductivity of solutions is the knowledge it gives regarding the degree of dissociation of the dissolved substance. The conductivity of an electrolyte is due entirely to the ions it contains and is directly proportional to the number of ions per cubic centimeter. Most salts are completely dissociated in very dilute solutions, and therefore the molecular conductivity of such solutions is not increased by further dilution. Call this value μ_0. Then if μ denotes the molecular conductivity of a more concentrated solution of the same salt, the relative dissociation in this solution is,

$$\alpha = \frac{\mu}{\mu_0}.$$

Express results by means of a curve, using values of μ for ordinates and the corresponding values of $\frac{1}{m}$ (= number of cubic centimeters containing 1 gram molecular) as abscissæ.

CHAPTER V

THE WHEATSTONE BRIDGE (*Continued*)

61. The Slide Wire Bridge with Extensions.—The measurement of resistances by the slide wire bridge can be made with more precision by using a longer bridge wire. The uncertainty in locating the balance points probably will be about the same, but since the distance, $a' - a''$, between the two balance points is increased, the percentage error will be less.

As it would be inconvenient to have the apparatus much over a meter in length, and as only the middle portion of the

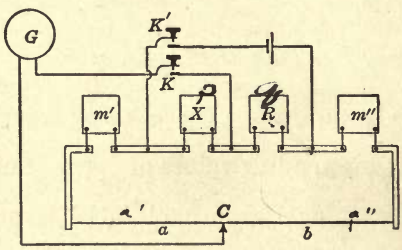


FIG. 38.—Bridge with extensions, m' and m''.

bridge wire is used in making careful measurements, the effective length of the bridge wire is increased by adding a resistance at each end. These extensions may consist of known lengths of wire similar to that used for the bridge wire—or any two equal resistances may be used and their equivalent lengths determined experimentally by the method shown below. The meter of wire provided with a scale then becomes only a short portion along the middle of the total length of the bridge wire. While this arrangement makes possible a greater precision of measurement, it also lessens the range of

the bridge, as only those balances which fall on this limited section of the wire can be read.

The extensions are placed in the outside openings on the back of the bridge, between the ends of the bridge wire and the battery connections. They should be nearly equal. Let m' and m'' denote the number of millimeters of bridge wire having the same resistances as each extension, respectively, and let L denote the total length of the bridge wire including both of its extensions.

With the resistance, x, to be measured, and the known resistance, R, in the middle openings of the bridge, as shown in Fig. 38, the first balance point is found. The reading on the scale at this point will be called a'. Then

$$\frac{x}{R}=\frac{m' + a'}{m'' + b'}=\frac{m' + a'}{L - (m' + a')} \tag{1}$$

Exchanging the positions of x and R, and calling the scale reading at the new balance point a'',

$$\frac{x}{R}=\frac{L - (m' + a'')}{m' + a''} \tag{2}$$

And by the additions of proportions,

$$\frac{x}{R}=\frac{L + (a' - a'')}{L - (a' - a'')}=\frac{L + d}{L - d} \tag{3}$$

It is evident that this arrangement reduces the range of the bridge, for only those values of x can be measured which are near enough equal to R to give balance points on the scale. But what is lost in range is more than made up in the greater precision of measurement.

Dividing out the fraction in (3) gives

$$\frac{x}{R}=1 + 2\frac{d}{L}+ 2\frac{d^2}{L^2}+ 2\frac{d^3}{L^3}+ \tag{4}$$

All the terms after the second are negligible if d is small in comparison with L, so that,

$$x = R + 2R\frac{d}{L} \tag{5}$$

20 → 40 $\frac{41.8}{3780}$

The only part of x, then, which is measured by the bridge is the second term, and a small error in it will only slightly affect the computed value of x.

62. To Find the Length of the Bridge Wire with Its Extensions.—The total length of the bridge wire, including the extensions at each end, can be determined as follows. A good resistance box, P, is used in place of the unknown resistance x shown in the figure of the preceding section. Then with both extensions connected in the bridge, the values of P and R are adjusted to bring the balance point near one end of the scale. Let a' denote this scale reading, corrected if necessary by the calibration curve for this wire when used as a simple bridge. Then

$$\frac{P}{P+R} = \frac{m' + a'}{m' + c + m''} = \frac{m' + a'}{L}$$

where c denotes the original length and L the total length of the bridge wire.

Exchanging P and R, the balance falls near the other end of the wire and

$$\frac{R}{P+R} = \frac{m' + a''}{L}$$

.50 .76 8

By subtraction,

$$\frac{R - P}{P + R} = \frac{a'' - a'}{L}$$

whence

$$L = \frac{P + R}{R - P}(a'' - a').$$

This method may also be used to determine whether there is any extra resistance in the straps and connections at the ends of the usual meter of bridge wire.

Problem.—In calibrating a bridge wire it was found that for P = 100, and Q = 900 ohms, the balance point fell at 95 on the scale; while for P = 900 and Q = 100 the balance point was at 903. What is the effective length of the bridge wire? Ans. L = 1010 mm.

63. To Calibrate the Slide Wire Bridge with Extensions.—The formula deduced for this method in eq. 5 above works very well as long as x and R are nearly equal; but several errors may occur in its use, the principal of which are:

1. Using a wrong value for L.
2. Neglecting all the terms containing d in powers higher than the first.
3. Errors in the determination of d, due to non-uniform bridge wire, scale errors, etc.

The method of calibration described below corrects for all of these errors at once by finding a correction to be added to the observed value of d, which will give to $\left(1 + 2\frac{d}{L}\right)$ the true value of $\frac{x}{R}$.

With the bridge set up as shown in Fig. 38, with the extensions in place, and two good resistance boxes, P and Q in place of x and R, we have,

$$P = Q\left(1 + \frac{2d}{L}\right) \qquad \text{(A)}$$

and solving for d gives,

$$d = \frac{L}{2Q}(\mathrm{P} - Q) \qquad \text{(B)}$$

This is the value that d must have in order that (A) shall give the correct values of the resistances being measured.

Starting with P and Q each 1000 ohms, the value of d should be zero. Then increasing P by successive small steps the corresponding observed values of d can be determined. These observed values of d will not agree with those computed from (B) above, and therefore if used in (A) will not give the correct values for P. This is because of the errors noted above. It

is therefore necessary to add to the observed length of bridge wire, d, a certain amount, c, such that

$$d + c = d'$$

and this corrected value, d', is what should be used in eq. 5 above.

In the present case where P and Q are known, the values of d' are computed from eq. (B), while the corresponding values of d are observed on the bridge wire. The differences give the values of c, and a calibration curve can be drawn, as shown, which will give the correction to be used at each point. The correction increases rapidly with d owing to the increasing importance of the second error noted above.

In case d is not zero when P and Q are nominally equal it means that one is really a little larger than the other. Let d_o denote this value of d. This can be reduced to zero by adjusting the value of Q; or if it is more convenient, d_o may be subtracted from each value of d throughout this calibration, as a sort of zero correction.

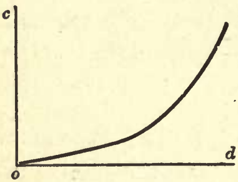


FIG. 39.—Calibration curve.

The data may be recorded as below.

TO CALIBRATE BRIDGE NO. .... WITH EXTENSIONS NO. .... AND NO. ...

P	Q	a'	a''	d $=a'-a''$	d' Eq. B	c $=d'-d$

64. Advantages of the Double Method.—When it is desired to use the slide wire bridge with some degree of precision several precautions are necessary in order to avoid the principal errors. Prominent among these are the effects due to thermal elec-

tromotive forces in the galvanometer circuit, to balance which it is necessary to set the sliding contact on the wire at a point somewhat to one side of the true balance point to obtain zero deflection of the galvanometer. If the scale is displaced endwise with respect to the wire, or if the index from which the readings are taken is not exactly in line with the point at which contact is made on the wire, the effect is much the same.

Let a' denote the observed reading on the scale, and $a' + f$ the true balance point, where f denotes the displacement of the reading due to the causes noted above. The actual value of f is unknown but it is constant in amount and sign, at least while one set of readings is being taken. For a balance of the bridge with x and R in the positions shown in Fig. 27, we have

$$\frac{x}{R} = \frac{a' + f}{c - (a' + f)}$$

where c is the total length of the bridge wire expressed in the same units as a' and f—usually in millimeters.

Exchanging x and R gives a new balance at a'', and

$$\frac{x}{R} = \frac{c - (a'' + f)}{a'' + f}$$

Combining these two expressions by the addition of proportions gives

$$\frac{x}{R} = \frac{c + (a' - a'')}{c - (a' - a'')} = \frac{c + d}{c - d}$$

It is seen that f has been eliminated by this double method and the only measured quantity appearing in the final expression is d, the length of the wire between the two *observed* balance points. The value of c should be determined by the method given in Article 62 as this may be greater than the meter of bridge wire because of the added resistance of the copper straps and the connections at each end of the wire. The "bridge wire" really includes all of the resistance from A to D. However, if d is small a slight uncertainty in the value of c will pro-

duce a negligible error in the computed value of x. This means that R should be taken as near to the value of x as is convenient.

65. The Best Position of Balance. (*a*) *Simple Bridge.*—The formula deduced in Article 49 for the value of a resistance measured by the simple slide wire bridge is

$$x = R\frac{a}{c - a} \tag{A}$$

Suppose that the value of a that is read on the scale and put into this formula is too large by F_a *mm.* This will make the computed value of x too large by F_x, say. It is required to find the quantity F_x. Let the relationship between F_x and F_a be denoted by m.

then $$F_x = m\,F_a \tag{B}$$

and it remains to find m.

From B, $$m = \frac{F_x}{F_a}.$$

F_a is a small part of a, and F_x is the corresponding small part of x. In the notation of the calculus this would be written,

$$m' = \frac{dx}{da}$$

and if F_a is small, $m' = m$, whence B becomes

$$F_x = F_a\frac{dx}{da} \tag{C}$$

From A,

$$\frac{dx}{da} = R\frac{(c - a) + a}{(c - a)^2} = R\frac{c}{(c - a)^2}.$$

and

$$F_x = F_a\frac{Rc}{(c - a)^2} \tag{D}$$

But the actual error made in finding the value of x is not of as much importance as the relative error. It is evident that an error of one ohm in a total of ten ohms is a very different thing

from an error of one ohm in a thousand ohms. The relative error is the ratio of the actual error to the total quantity measured. Thus from (A) and (D) the relative error, e, is

$$e = \frac{F_x}{x} = F_a \frac{c}{(c - a)a}.$$

From this it appears that even the relative error is not the same for the same error in reading, but it depends upon the value of a. Examining this expression for a minimum value of e

$$\frac{de}{da} = \frac{-c(c - 2a)}{((c - a)a)^2} F_a = 0.$$

This is satisfied if $(c - 2a) = 0$.

Thus in reading the value of a, a given error (say 1 mm.) will produce the least effect on the computed value of x when the balance point comes at the middle of the bridge wire.

(*b*) *Double Method*

The above discussion applies to the simple slide wire bridge. When the double method is used the formula for the resistance being measured is

$$x = R \frac{1000 + (a' - a'')}{1000 - (a' - a'')}$$

as deduced in Article 51.

Writing this as

$$x = R \frac{c + z}{c - z}$$

it can be shown in the same way that a given error in measuring z will have the least effect upon the computed value of x when $z = 0$, that is, when both a' and a'' are at the middle of the bridge wire. In this case

$$\frac{dx}{dz} = R \frac{2c}{(c - z)^2}$$

and

$$e = F_z \frac{2c}{(c - z)(c + z)} = \frac{2c F_z}{c^2 - z^2}$$

which evidently is a minimum when $z = 0$.

66. Sources of Error in Using the Slide Wire Bridge.—These may be summarized as

I. Errors in setting, due to
- *a*. Thermal currents.
- *b*. Contact maker not in line with index.
- *c*. Non-uniform wire or scale.
- *d*. Ends of wire and scale not coincident.

II. Errors in reading.
- *e*. The position of balance.
- *f*. True value of R, loose plugs, etc.

The effect of a and b can be eliminated by using the double method, as explained in Article 64.

The only way to avoid the effect of c or d is by calibration of the bridge wire and correcting all readings.

The error in reading the position of the index after a balance has been found is often greater than the uncertainty of the setting. In the preceding section it was shown that this error, which is about the same for all parts of the scale, has the least effect on the computed value of x when the reading is near the middle of the bridge wire.

The error in the resistance coils of a good box is very small. However, the value of R read from the box and used in computations may be very different from the actual resistance of the experiment. If some of the plugs are loose, or make poor contact because of dirt or corrosion, the resistance may be considerably increased. Moreover, the resistance actually used in the bridge includes all the connections and lead wires used to join the box to the bridge. In the same way the resistance measured includes the lead wires and connections.

67. The Direct Reading Bridge.—The ordinary slide wire bridge used with extension at each end of the bridge wire can easily be made into a direct reading bridge. Its peculiarity lies in its calibration and the adjustment of the extensions. When a large number of resistances are to be measured the double readings and consequent calculations are too slow a method. It may then be better to adjust the extensions to

make the total length of the bridge wire 40 meters, or each extension equivalent to 19.5 meters of the bridge wire. With this length, and adjusted to bring the middle point at 50 on the scale, the balance point will fall at 51 for values of x larger than R by 0.1 per cent., and 1 cm. further from the center for each additional 0.1 per cent. between x and R. Only a single balance is obtained and it is then very easy to read from the scale the percentage by which x is larger than R. This relation, which is exact at the center, departs more and more from the scale reading as the balance point falls further from the center. The slight corrections needed are readily obtained from the calibration curve.

For the calibration of the bridge two well adjusted resistance boxes are inserted in the back openings of the bridge. By setting each box at 5000 ohms the middle point of the total bridge wire, including the extensions, is located. One or both of the extensions should be adjusted to bring this point to 50 on the scale. If now one of the boxes, say Q, is increased by 5 ohms, a new balance point is obtained; and this point corresponds to values of Q which are 0.1 per cent. larger than P. Q is increased by 5 ohm steps, giving balance points corresponding to 0.2 per cent., 0.3 per cent., etc. These results may be plotted as a calibration curve, but it is more legible to plot only the differences between the correct readings and those actually read from the scale. The battery should be reversed and the mean of the two readings employed in plotting the curve.

In using the bridge to measure an unknown resistance a single balance point is obtained (or the mean of two if reversing the battery makes any change). This reading gives at once the approximate percentage by which x is larger than R. A glance at the calibration curve gives the correction to be added to obtain the correct percentage. A short multiplication gives the results in ohms.

68. Measurement of Resistance by Carey Foster's Method. —One of the most exact methods for comparing two resist-

ances is the one devised by Prof. Carey Foster of England. The Wheatstone bridge is arranged in the same manner as was used for the slide-wire bridge with extensions, except that now the extensions become the resistances to be compared.

Thus in the figure let S be the resistance that is to be compared with R. These two are placed in the bridge as shown, being connected together by the bridge wire. The other arms of the bridge, P and Q, become merely ratio coils and may have any value although they must be nearly equal to each other in order that the balance may fall on the bridge wire,

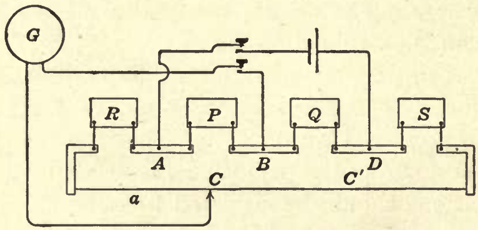


FIG. 40.—Carey Foster bridge, comparing R and S.

and for the most sensitive arrangement *all the arms* of a Wheatstone bridge should be nearly equal. For the greatest accuracy R and S should each be in a constant temperature oil bath.

Let the balance point be found by moving the contact C along the wire until a point is reached for which there is no deflection of the galvanometer. Let a' be the scale reading at this point. It is immaterial whether this scale extends the entire length of the bridge wire or not.

Now let R and S be exchanged with each other. This will make no difference in the total length of the extended bridge wire, ACD. But if the resistance of S is less than that of R the new balance point will not fall at the same point as before, for it will be necessary to add to S enough of the bridge wire to make the part AC the same as before. The resistance of AC in the first instance is

$$R + a'p = \frac{P}{P+Q}\mathsf{T}.$$

After R and S have been exchanged the resistance of AC' is

$$S + a''p = \frac{P}{P+Q}\mathsf{T}.$$

where T denotes the total resistance of ACD. Equating these two expressions

$$R + a'p = S + a''p,$$

and

$$S = R - (a'' - a')p$$

where p is the resistance per unit length of the bridge wire.

69. To Determine the Value of p.—The resistance per unit length of bridge wire can be readily determined by the same arrangement as shown above. Let R and S be two resistances differing by a small, but very accurately known, difference. For example, $R = 0$ and $S = 0.500$ ohm could be used if these values are definitely known. Or, $R = 1.0$ ohm and S composed of a one ohm coil in parallel with a ten ohm coil would give a smaller difference which could be used, *provided* again the values of the original coils are really known. It is not essential that the resistances of the coils be precisely one ohm, or ten ohms, but it is necessary that the actual values be known in order that the small difference between them can be accurately computed.

Then since $$R - S = (a'' - a')p$$
it follows that

$$p = \frac{R - S}{a'' - a'}$$

70. Temperature Coefficient of Resistance.—An interesting application of the above methods for the accurate measurement of resistance is found in the determination of the temperature coefficient of resistance for various metals and alloys. When the temperature of a conductor is varied its resistance generally changes also, the usual relation being that an increase of temperature is accompanied by an increase in resistance. The amount of this increase will evidently be greater for a large resistance than for a small one, and it is conveniently

expressed as a certain fraction of the original resistance. Thus in symbols

$$\frac{dR}{dt} = a_t R_t \qquad (1)$$

where R_t is the resistance of the conductor at some one definite temperature, and the coefficient a_t denotes the fraction by which each ohm of R_t changes per degree change in temperature.

If the change in resistance per degree is constant over a considerable range of temperature, then

$$\frac{R_1 - R_2}{t_1 - t_2} = a_t R_t$$

and

$$a_t = \frac{R_1 - R_2}{t_1 - t_2} \cdot \frac{1}{R_t}$$

It should be noted that this value of a applies to the whole temperature range, $t_1 - t_2$, but its numerical value depends upon the value, R_t, that is taken as the true resistance of the coil under consideration.

70A. Determination of the Temperature Coefficient.—In this experiment five coils of different metals are arranged in an oil bath where the temperature can be raised as desired. A very convenient arrangement is to use an electric water heater to hold the oil bath. With a suitable resistance in series this can be easily warmed and maintained at the desired temperatures. Starting at room temperature the resistance of each coil is carefully determined. A Wheatstone bridge box gives a convenient and accurate method for resistances having large temperature coefficients like copper and iron. For alloys like German silver and manganin it is better to use a more delicate method, such as the Carey Foster bridge.

After the resistances of the coils have been obtained at room temperature the bath is warmed ten or fifteen degrees, and when things have become steady at the new temperature the resistances are again measured. In the same way the

resistances are determined at five or six different temperatures, and the results plotted as a temperature-resistance curve for each coil. The slope of this curve is dR/dt of Eq. (1). The resistance of the coil at 20° C. can be read from the curve. Then

$$a_{20} = \frac{1}{R_{20}} \frac{R_1 - R_2}{t_1 - t_2}$$

where the values of R_1 and R_2 are obtained from the curve at points corresponding to the temperatures t_1 and t_2. Exterpolating the curve back to 0° C., the value of R_0 can be obtained from the curve in the same way, and,

$$a_0 = \frac{1}{R_0} \frac{R_1 - R_2}{t_1 - t_2}$$

70B. Use of the Temperature Coefficient.—If 20° C. is taken as the standard temperature then

$$a_{20} = \frac{R_1 - R_2}{R_{20}(t_1 - t_2)}$$

In using this formula for finding the resistance at any desired temperature t_1, we have

$$R_1 = R_2 + a_{20}R_{20}(t_1 - t_2).$$

and further, if t_2 is also 20° C.

$$R_1 = R_{20}(1 + a_{20}(t_1 - 20))$$

If 0° C. is taken as the standard temperature,

$$R_1 = R_0(1 + a_0 t_1)$$

CHAPTER VI

MEASUREMENT OF CURRENT

71. Hot Wire Ammeter.—When an electric current flows through a wire one of the most noticeable effects is that the wire becomes warmed. The direct result of this is an increase in the length of the wire. It thus becomes possible to measure the value of the current in terms of the change in length of such a wire, and some ammeters are constructed on this principle.

72. The Weston Ammeter.—In addition to heating the wire the current produces a magnetic effect in the surrounding space. This is manifest by its action upon a magnetic needle near it, or by the force which the wire itself experiences when in the magnetic field of a magnet or another current. This effect furnishes the basis for defining the value of unit current (see Introduction, Article 4) and the precise relation between the current and the force acting upon the wire is fully worked out in Chapter XI. From this it follows that a loop of wire carrying a current in a magnetic field will tend to turn in a definite direction according to the direction of the current.

In the Weston ammeter the current passes through a coil of many turns of fine wire wound on a rectangular form. This coil is mounted on jeweled bearings and can turn in the magnetic field between the poles of a strong permanent magnet. It is held in a definite position by a spiral spring at either end. When a current is passed through the coil it turns until the torque of the springs is sufficient to balance the couple due to the forces between the current and the magnetic field. A pointer attached to the coil moves over the scale and indicates the angle turned through; the scale is not graduated in degrees,

but in terms of the current required to produce the deflection. The scale is thus direct reading and gives the value of the current in amperes.

In ammeters for measuring large currents a low resistance shunt is placed in parallel with the moving coil to allow only a moderate current through the latter. The scale is then graduated to read the value of the large total current through both the coil and its shunt. This arrangement is entirely similar, in principle, to the voltmeter and shunt described in Article 23, the moving coil system acting as a sensitive voltmeter.

73. The Weston Voltmeter.—The construction of a voltmeter is the same as an ammeter, except that instead of having a shunt in parallel with the moving coil there is a high resistance in series with the coil. The current which will then flow through the instrument depends upon the E.M.F. applied to the terminals of this resistance, and the numbers written on the scale are not the values of the currents in the coil, but are the corresponding values of the fall of potential over the resistance of the voltmeter. Therefore it is sometimes said that a voltmeter is really only a sensitive ammeter measuring the current through a fixed high resistance; while an ammeter is really only a sensitive voltmeter measuring the fall of potential over a low resistance.

74. Galvanometers.—The D'Arsonval type of galvanometer consists of a moving coil suspended between the poles of a permanent magnet something like the arrangement in an ammeter, but greater sensitiveness is secured by using a long, fine strip of phosphor bronze for the suspension in place of the jewel bearings. Such a galvanometer measures very small currents and is useful in measurements where the current is small or is made zero in the final adjustment. It can be used to measure larger currents by using a low resistance shunt, as in the ammeter; and it will serve as a voltmeter when used in series with a high resistance.

Since the scale from which is read the deflection of the

galvanometer usually is divided into millimeters, it will be necessary to calibrate it in order to read the value of the current. A setup for determining the figure of merit can be used (see Figure 21) and deflections corresponding to different values of the current can be observed. A curve can then be plotted between deflections and currents, and from this curve can be read the value of the current corresponding to any deflection.

75. The Tangent Galvanometer.—In the instruments heretofore described it has been supposed that the scale was graduated to read the value of the current directly in amperes, or that it could be calibrated so to read. But nothing has been said as to how the value of a current can be expressed in terms of the ampere as defined in the Introduction. The tangent galvanometer furnishes the means for establishing the value of a currnet in terms of the C.G.S. unit. The method is readily understood and formerly was the principal method for determining the absolute value of a current. Other methods (see Chapter VII) now offer more precise measurements, but the tangent galvanometer is as accurate as it ever was and it still holds sufficient historical interest to warrant its description.

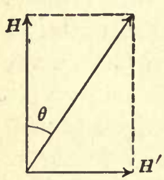


Fig. 41.—Resultant of two magnetic fields.

The tangent galvanometer consists of a coil of relatively small cross section and large diameter. At the center is a short magnetic needle, suspended by a fine silk or quartz fiber and carrying either a long pointer or a mirror for use with a telescope and scale. When hanging freely the needle will point north and south in the magnetic meridian. The large coil should stand vertically in this same meridian. When a current is passed through the coil the magnetic field at the center of the coil due to this current will be directed east and west. The resultant field due to the combination of this field with the original field H, of the earth, will be in a direction intermediate between the two and making an angle θ, say, with

the direction of the latter. The needle, when free to turn, will take up this resultant direction; therefore the angle θ is determined by observing the change in the scale reading when the needle is deflected.

The intensity of the magnetic field at the center of a coil of n turns is, from Chapter XI,

$$H' = \frac{2\pi In}{r},$$

where r is the mean radius of the coil.

Since the coil of the tangent galvanometer is so placed that H' is at right angles to the earth's field, H, the angle which the resultant of these two makes with the latter is given by the expression,

$$\tan\,\theta = \frac{H'}{H} = \frac{2\pi In}{rH},$$

whence,

$$I = \frac{Hr}{2\pi n}\tan\,\theta,$$

where I denotes the value of the current in C.G.S. units and θ is the corresponding deflection of the needle.

76. The Coulometer.—Another effect of an electric current is manifest when the current passes through an electrolyte, such as a solution of copper sulphate in water. At the point where the current leaves the solution copper is deposited, and the amount of copper so deposited is directly proportional to the current and the time which it flows, that is, to the number of coulombs of electricity that have passed through the electrolyte. With proper precautions this gives a good method for measuring current, and indeed the value of the International Ampere is fixed in terms of the amount of silver deposited per second in a silver nitrate solution.

When the copper coulometer is used the electrodes should be of copper, the best form being obtained by winding a meter of large copper wire into a loose spiral that will just fit inside the glass jar containing the solution. Another spiral wound

smaller and suspended within the first one serves as the cathode. All of the copper surface should be clean and never touched with the fingers. When ready to be used the current should be passed through the coulometer for a sufficient time to deposit a uniform layer of new copper upon the smaller spiral, which is always used for the cathode. During this preliminary run the current should be adjusted to the value it is desired to measure.

As soon as the current is stopped the spiral cathode is removed from the solution and thoroughly rinsed in distilled water, then dipped in alcohol to remove the water. The alcohol evaporates readily when the coil is gently swung in warm air. When completely dry the cathode is carefully weighed.

When ready to begin the run the cathode is placed in position in the solution, and at a given moment the circuit is closed, thus starting the current at a known time. The current should be maintained as constant as possible for an hour and then stopped at another known time. The cathode is removed, washed, dried and weighed as before. The gain in weight gives the amount of copper deposited, from which the average value of the current can be computed. Since one coulomb will deposit 0.0003283 gram of copper, the current is given by the expression,

$$I = \frac{M}{0.0003283t},$$

where M denotes the mass of copper deposited in t seconds.

77. The Kelvin Balance.—The Kelvin balance is an accurate semi-portable instrument for the measurement of current. There are six flat coils placed horizontally and through which the current passes in series. Two of these are carried on a balanced beam, one at either end, while above and below each of these movable coils is one of the fixed coils. The diagram shows the relative positions of the coils. The movable coil *cd* is shown in a vertical section through a diameter, while the dotted lines indicate the magnetic field in the same plane due to the fixed coils *ab* and *ef*. It will be seen that at *c* the field is

horizontal and directed to the right. If the current in the coil *cd* is flowing into the paper at *c*, then this portion of the circuit will be urged downward. At the other side of the coil the direction of the field is to the left, and the current at *d* is flowing out of the paper. Hence this side of the coil will also be urged downward. The same is true for all parts of the coil *cd*, and therefore the coil as a whole is urged downward with a force proportional to the product of the current it carries and the intensity of the magnetic field in which it moves (see Chapter XI). The latter is proportional to the current in the fixed coils, therefore the downward pull is proportional to the square of the current through the coils. The action at the other end of the balance is the same, but the direction of the

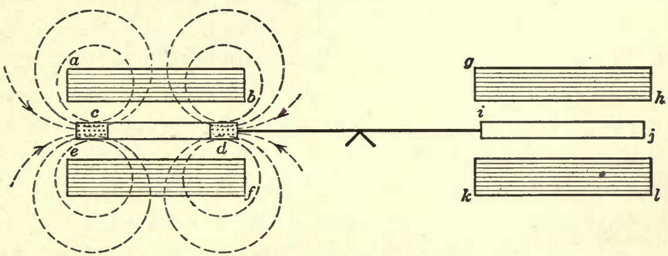


FIG. 42.—Coils of the Kelvin balance

current through the coils is such that the movable coil *ij* is urged upward. Thus this effect is added to the other.

To restore the balance a sliding weight is drawn along a graduated beam until the movable coils again stand in their original position midway between the fixed coils. This position is indicated by a short scale at the end over which moves a pointer. The position of the weight on the beam is read from a scale of equal divisions, and as shown above this is proportional to the square of the current. To obtain the value of the current in amperes the square root of this reading is multiplied by the constant corresponding to the particular weight used. There are four such weights and the constants are 0.5, 1, 2,

and 4, respectively, for most balances. In the centi-ampere balance the result will then be given in centi-amperes.

One other matter must be noticed. When the sliding weight is placed on the beam at the zero end of the scale it is necessary to place an equal counterpoise in the pan at the other end. If this does not establish a complete balance there is a small brass flag carried by the moving system which can be turned so as to throw more weight to one side or the other as may be required to restore the balance.

78. The Electrodynamometer.—The e-lec″tro-dy″na-mom′-e-ter is an instrument for measuring currents. It consists essentially of two vertical coils, one fixed in place, and the other free to turn about the vertical axis common to both coils. Sometimes the movable coil is outside the other, as in the Siemens type: in other forms the movable coil is within the fixed coil. In either case when a current flows through the movable coil it tends to turn in the same manner as the coil of a D'Arsonval galvanometer. But in the electrodynamometer the magnetic field is not due to a permanent steel magnet, but is produced by the current flowing in the fixed coil. Thus the deflection depends upon the current I in the fixed coil as well as upon the current i in the movable coil: and the resulting deflection is given by

$$iI = A^2D$$

where A^2 is a constant including all the factors relating to the size and form of the coils, etc., and also including the restoring couple of the suspension.

If the same current flows through both coils in series,

$$I^2 = A^2D \qquad \text{and} \qquad I = A\sqrt{D}$$

In the Siemens electrodynamometer the coil is brought back to its resting point by the torsion of a helical spring. D is the number of divisions of the scale which measures the amount of this torsion.

When a coil carrying a current is suspended in a magnetic

field, *e.g.*, the earth's field, it tends to turn so as to add its magnetic field to the other. If the electrodynamometer is set in such a position that the earth's field is added to its own, evidently the deflection will be increased by a corresponding amount. If the two fields were opposed to each other the deflection would be lessened. This effect can be eliminated by turning the instrument so that the plane of the movable coil is east and west.

79. Calibration of an Electrodynamometer.—In order to use an electrodynamometer for the measurement of a current it is necessary to know the value of the constant, A, or, what is better, to have a calibration curve. Such a curve is obtained by joining the instrument in series with a good ammeter or a Kelvin Balance, and observing corresponding readings on the two instruments when carrying the same current. The curve is carefully plotted, using currents as ordinates and the corresponding deflections for abscissæ. This gives a horizontal parabola passing through the origin, and from this curve the value of the current corresponding to any deflection can be read. With such a curve, the dynamometer becomes a direct reading ammeter. The deflection is independent of the direction of the current through the instrument, and therefore it can be used with alternating currents as well as with direct currents. Reversing the direction of the current through one coil only, however, will reverse the direction of the deflection.

80. For the measurement of current by means of a standard cell see Chapter VII.

CHAPTER VII

POTENTIOMETER METHODS

81. Potential Differences.—Any hill may be called up, or down, according to the direction in which one is going. If after traversing several paths one returns to the starting-point, there will have been as much down hill as up hill in the entire journey.

If going up hill is called positive, and down hill is called negative, the total height, up and down, that one has been raised is zero. The same thing would be true if down hill had been called positive and up hill called negative, but it seems more natural to use the words in the former sense.

In this sense, then, let us notice that when we follow a stream down hill, that is, in the direction in which the water flows, our change in level is negative, and our final position is lower than that at which we started.

82. Kirchhoff's Two Laws.—Kirchhoff has enunciated in the form of two "Laws" the principle by which it is possible to investigate the distribution of current and potential in very complicated cases; cases where there are any number of cells connected by a network of conductors in any way whatever.

Law I.—If any number of conductors meet at a point, and if all currents flowing to the point be considered positive, and all currents flowing from the point be considered negative, then when the currents have reached their steady values the algebraic sum of all the currents meeting at the point must be zero, or,

$$I + I' + I'' + \ldots\ldots = 0.$$

because it is not possible for a charge to accumulate indefinitely at any point.

Law II.—Let us suppose that there is such a network of conductors as imagined above, with cells of various E.M.F's., in the different branches of this network.

If we imagine ourselves to start from any point in this network and to make a circuit through the conductors back to our starting-point, we shall have passed through conductors of various resistances, shall have passed through various cells whose E.M.F's. are directed either in the direction we are going, or the opposite, and shall have found various currents, some with us and some against us.

As long as we are passing along a single conductor r, to which there are no outlets, the current has some fixed value i; but on passing a point where two or more conductors meet we may find a different current i', which will remain constant over the next piece of resistance r' up to the next place where two or more branches meet. We can thus divide our circuit into portions of resistances r, r', r'', etc., along each of which will be a current i, i', i'', etc., respectively.

If we call each product, ri, negative when we are following the circuit down the fall of potential, i.e., with the current, and positive when we are going against the current, and if we call each E.M.F. that we encounter positive when we pass up the potential difference, i.e., in the direction in which it tends to send a current, and negative when we pass down the potential difference, then *in the complete circuit* we must have

$$ri + r'i' + r''i'' + \text{etc.} + e + e' + e'' + \text{etc.} = 0$$

where e, e', etc., are the various E.M.F's. that we pass, and which may or may not occur in the resistances r, r', r'', etc., respectively.

Briefly, then, this law states that the *sum of all the potential differences in a closed circuit is zero.* This must be true since in tracing out a complete circuit we return to the starting point, and therefore to the same potential at which we started.

83. Illustrations of Kirchhoff's Second Law. *For a Simple Circuit.*—Consider a circuit consisting of a cell E, joined in series with a resistance R. Starting at A and going around the circuit counter clockwise we have a fall of potential RI in the part AB. In passing from B on to the starting-point A, there is the further resistance r of the battery, and therefore a further fall of potential of rI. The only E.M.F. in this circuit is that of the battery and it tends to send a current the way we are tracing out the circuit.

For this case, then, the law gives the relation

$$-RI - rI + E = 0 \quad \text{or} \quad E = RI + rI.$$

For the Wheatstone Bridge.—When the bridge is balanced

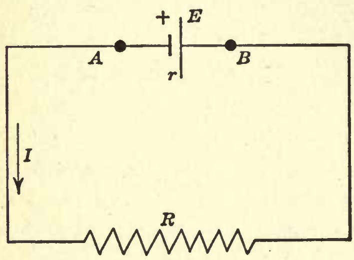


Fig. 43.

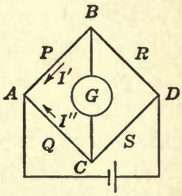


Fig. 44.

no current flows through the galvanometer. Then applying Kirchhoff's second law to the circuit $ABCA$ gives,

$$PI' - QI'' = 0,$$

since there is no E.M.F. in this circuit and no fall of potential in the galvanometer branch BC.

Similarly for the circuit $BDCB$,

$$RI' - SI'' = 0$$

Eliminating the I' and I'' gives the relation

$$PS = QR.$$

84. Proof of Kirchhoff's Second Law.—In order to fix ideas let us consider the case shown in the figure, and which may be extended to as many branches as we please. The figure shows a single net, or circuit, from a network of conductors through which currents are flowing. We will assume that the currents flow as indicated by the arrows; should the value of any current be found negative in the final solution it will mean simply that the actual current flows in the opposite direction.

Starting at any point we please, say D, and tracing out the circuit in either direction, say $D.E.A.$, etc., we can express

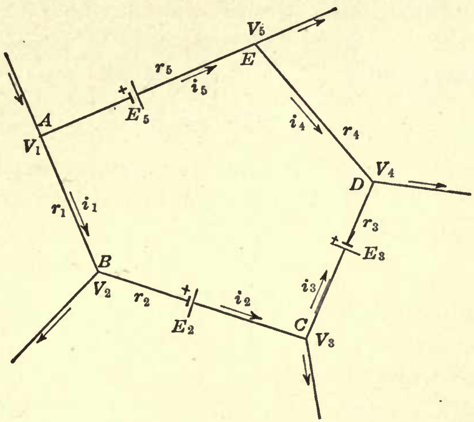


Fig. 45.

the differences of potential over each branch in terms of the resistances and E.M.F's. Since the current is taken as flowing from E to D, E must have the higher potential, and therefore the rise of potential from D to E is properly expressed by $+ r_4 i_4$.

Therefore along this branch of the circuit the change in potential is,

$$V_5 - V_4 = + r_4 i_4.$$

For the next portion of the circuit, EA,

$$V_1 - V_5 = + r_5 i_5 + E_5,$$

where in addition to the rise of potential, $r_5 i_5$, there is the further rise of potential E_5 as we pass through the cell from the negative to the positive electrode. The internal resistance of the cell is included in r_5, and E_5 denotes the total E.M.F. of the cell as measured by the condenser method or by the potentiometer.

In the next branch

$$V_2 - V_1 = - r_1 i_1,$$

where the negative sign indicates that there is a *fall* of potential along the conductor AB in the direction in which we are tracing out the circuit.

For the branch BC,

$$V_3 - V_2 = - r_2 i_2 - E_2,$$

because there is a drop of potential as we pass through the cell in addition to the fall along the conductor.

In the last branch

$$V_4 - V_3 = - r_3 i_3 + E_3$$

since the cell in this branch is set so that we pass through it from the negative to the positive electrode. It is to be observed that the sign to be given E has nothing to do with the direction in which the current is supposed to be passing through the cell. In such a network of conductors and cells it may often happen that the current will flow through some of the cells in the direction opposite to that which it would flow if such cells were acting alone.

Adding the equations for the complete circuit gives

$$(V_5 - V_4) + (V_1 - V_5) + (V_2 - V_1) + (V_3 - V_2) + (V_4 - V_3)$$
$$= r_4 i_4 + r_5 i_5 + E_5 - r_1 i_1 - r_2 i_2 - E_2 - r_3 i_3 + E_3 = 0$$

since the V's mutually cancel themselves.

This equation is seen to agree with the statement of the law as given in Article 82 above.

85. The Potentiometer Method.—The most exact method for measuring the E.M.F. of a cell is that known as the

potentiometer method. The principle employed will be readily understood by referring to Fig. 46. AD represents a meter wire and scale similar to a slide wire bridge but of much higher resistance. The ends of this wire are connected to the battery B, the E.M.F. of which is greater than that of any cell to be measured. C is the sliding contact, and between A and C can be included any fraction of the fall of potential along the wire from zero to the full amount. A and C are joined together through a shunt circuit containing the galvanometer, a key and the cell whose E.M.F., E, is to be measured.

By moving C along AD a point can be found for which there is zero deflection of the galvanometer when K is closed. Let a' be the distance from A to this point; the fall of potential along this portion of the wire is $a'pi$, where p is the resistance of unit length of the wire and i is the current flowing through it from the battery at B. Writing Kirchhoff's law for the galvanometer circuit gives,

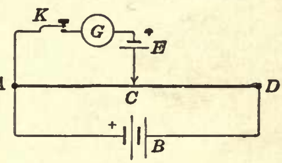


FIG. 46.—Slide wire potentiometer.

$$E' - a'pi = O \qquad \text{or} \qquad a'pi = E'$$

since there is no current and therefore no fall of potential in the galvanometer.

Now let another cell, whose E.M.F., E'', we may suppose is larger than E', be substituted for E. In order to obtain a balance and zero deflection it will be necessary to move C nearer to D. Let a'' be the reading of this position. Then by Kirchhoff's law, as before,

$$a''pi = E''$$

Dividing one equation by the other,

$$\frac{E'}{E''} = \frac{a'}{a''} \qquad \text{or} \qquad E' = a'\,\frac{E''}{a''}.$$

If E'' is known, E' can be computed from this proportion, being simply a constant (E''/a'') times the scale reading, a'. By making the wire two meters in length and adjusting the current to give a fall of potential of just two volts between A and D, the millimeter scale becomes direct reading, 1 mm. corresponding to one millivolt. With this arrangement the E.M.F. of any cell can be read from the scale as soon as the galvanometer balance is obtained.

86. The Resistance Box Potentiometer.—In the resistance box potentiometer the wire AD of the preceding section is replaced by two similar and well adjusted resistance boxes; and instead of actually moving C along the wire, resistance is transferred from AC to CD by changing the plugs while keeping the total resistance unchanged.

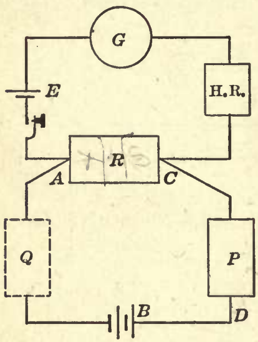


FIG. 47.—Resistance box potentiometer.

The arrangement is shown in Fig. 47. The cell to be measured is placed at E where it is in series with a sensitive galvanometer, a high resistance, HR, a key, and a resistance box R. Through the latter can be passed a small and constant current from the battery B. In Article 22 this R was the voltmeter, and the *current* through it was varied until the fall of potential, Ri, just balanced the E.M.F. of the cell. In the potentiometer method the *current is kept constant* and R is varied. Furthermore the galvanometer is a more sensitive indicator of when the balance has been secured.

The rest of the setup consists of a constant E.M.F. battery B, for supplying the constant current, and a resistance box P, which should be identical with R for convenience. The total resistance used in both P and R should be kept equal to the total capacity of one box; and if the boxes are alike it is very easy to keep the sum at this value.

Set up and test the arrangement, using first an old cell for E

so as not to endanger a valuable standard by an accident or wrong connection. Keeping $R + P$ at the total amount in one box a balance should be obtained without much difficulty. When it is certain that everything is working correctly the cells to be measured may be substituted for the cell at E.

The fall of potential across the resistance R is, by Ohm's law,

$$V = Ri, \tag{1}$$

where i is the current from the auxiliary battery B. For zero deflection of the galvanometer this fall of potential must be adjusted to just equal and balance the E.M.F., E of the cell being measured. That is, zero deflection means

$$E = V, \tag{2}$$

or, from (1)

$$E = Ri. \tag{3}$$

When the known E.M.F., E', of another cell has been balanced by the proper adjustment of R, this relation may be expressed in the same way as before,

$$E' = R'i \tag{4}$$

Since both E' and R' are known this equation determines the value of the current i as

$$i = \frac{E'}{R'} \tag{5}$$

and if the current has been kept constant while R was varied (by also varying P so as to keep $P + R$ constant) and the battery B has not changed, then this value of i can be used for the value of the current in (3), giving,

$$E = E'\frac{R}{R'} \tag{6}$$

which expresses the E.M.F. of the cell to be measured in terms of known quantities.

A further simplification can be introduced as follows:

Set $R = 10{,}000\,E$, where E is numerically equal to the E.M.F. of the standard cell at its present temperature. Set P at its complementary value in order that later it will be convenient to keep $P + R$ constant. This, in general, will not give a balance, but another resistance, Q, may be introduced into the main circuit and adjusted to give the balance. Leaving Q at this value, the cell at E is replaced by one whose E.M.F. is desired and the balance obtained by adjusting R and P, keeping their sum constant as before. The resistance in R for a balance gives directly the E.M.F. of this cell. In other words, the factor E'/R' in (6) has now been made a round number ($= 0.0001000$) and the computatior consists merely in moving the decimal point four places to the left.

The data can be arranged as below

Use of Potentiometer

Name of cell	R	P	Q	E

87. The Potentiometer.—For purposes of instruction the three box potentiometer just described enables one to see clearly all parts of the setup, and as has been shown it is convenient to use. But if one has many measurements of E.M.F. to make, the mere changing of the resistances to obtain the balances becomes a laborious task. It is then desirable to have a quicker and simpler way of varying the resistances. This is accomplished in the first place by arranging the resistances on a dial over which turns an arm, instead of using a movable plug; and in the second place there is some automatic arrangement for keeping constant the total resistance in the circuit.

In a high resistance potentiometer the resistance R, Fig. 47, is arranged on the dial plan. As the arm sweeps over the dial,

thus varying the amount of resistance in R, another arm sweeps over the corresponding dial for P, but in the opposite direction; thus by a single movement of the hand the resistances in R and P may be varied while their sum remains constant.

In a low resistance potentiometer the arrangement is more as shown in Fig. 46, where the resistance of the main circuit, $BACDB$, remains unchanged. The portion ACD consists of about fifteen equal resistances, the first one being a long slide wire. In making a balance the contact C is moved along by a series of equal steps until the balance has been obtained to the nearest step. Then the final adjustment is made by moving the other galvanometer terminal from A along the slide wire until the balance is obtained. In this arrangement there are no variable contacts in the potentiometer circuit, the contacts at A and C both being in the galvanometer circuit where more or less resistance does not affect the position of balance.

An arrangement in which the various resistances are thus conveniently brought together in a single box is called a potentiometer.

88. Standard Cells.—In all measurements of E.M.F. with the potentiometer it is necessary to have a known E.M.F. which can be used as explained in Article 86 (E', Eq. 6), to standardize the value of the current through the potentiometer. Such a known E.M.F. is furnished by a standard cell, which is a primary battery set up in accordance with definite specifications so that it will possess a definite E.M.F. Such a cell is used as a standard of E.M.F., and is never expected to furnish a current. Since the E.M.F. of a cell depends upon the materials used in its construction, and not at all upon its size, standard cells are made small, both for economy of materials and for convenience in handling them. A simple form of a standard cell consists of a small Daniell cell of the porous cup form, set up with half saturated solutions of zinc sulphate and of copper sulphate. The copper electrode should be clean and the zinc electrode well amalgamated. This cell has an E.M.F. of about 1.08 volts.

89. The Weston Standard Cell.—The Weston standard cell was devised by Mr. Edward Weston. It has been the subject of much study and investigation, so that now it is possible for investigators in different parts of the world to set up such cells and know that they will have the same E.M.F. to within less than a ten-thousandth of a volt. In order to attain such accuracy it is necessary that the cells be set up in strict accordance with the specifications, using only the purest materials.

The cell is usually set up in an H-shaped glass vessel having dimensions of a few centimeters. At the bottom of one leg is placed pure mercury to form the positive electrode of the cell. Connection to this is made by a fine platinum wire sealed into the glass, the inner end being completely covered by the mercury. At the bottom of the other leg there is placed, similarly, some cadmium amalgam which when warm can be poured in like the mercury and then hardens as it cools. A second platinum wire through the glass at the bottom of the leg makes electrical connection with this electrode. The electrolyte is a saturated solution of cadmium sulphate, containing crystals of cadmium sulphate in order to keep the solution saturated at all times. The mercury electrode is protected from contamination by the cadmium in this solution by a thick layer of a paste consisting mainly of mercurous sulphate. Cadmium ions from the solution coming through this paste form cadmium sulphate, and only mercury ions pass on and come into contact with the mercury electrode. This paste is thus an efficient depolarizer. (See page 9.)

90. Use of a Standard Cell.—A cell set up as described above will have a definite E.M.F. which varies but slightly with the temperature. Its internal resistance is high, and therefore it would not be possible for it to furnish much current. In fact, any appreciable current drawn from the cell would polarize it somewhat, thereby decreasing its E.M.F. by an unknown amount and thus destroy the only value which the cell possesses. The depolarizer tends to restore the E.M.F. to its

original value, but the time required depends upon the amount of polarization.

Standard cells may be used to charge condensers to a known difference of potential, for in this case there is no steady current drawn from the cell and the transient current is not sufficient to cause an appreciable polarization. When used in connection with a potentiometer the cell should always be placed in series with a sensitive galvanometer. If it is necessary to reduce the deflection this should be done by means of a high resistance in series with the cell, instead of a shunt on the galvanometer which would still allow the large current to flow through the standard cell. When using the galvanometer the key should be lightly and quickly tapped so as to give a deflection of only a centimeter or two. This will indicate the direction of the current as clearly as a larger deflection and does not injure the standard cell.

91. Comparison of Resistances by the Potentiometer.—One of the accurate methods for comparing two resistances, particularly when these are not very large, is by means of a potentiometer. The two resistances to be compared are joined together in series with a battery and sufficient other resistance to ensure a steady current. This current should not be large enough to change the resistances by heating them. Other things being equal, it is desirable to have the fall of potential over each resistance about one volt. Let the two resistances be denoted by S and T; then the fall of potential over each will be SI and TI, respectively.

The actual measurements are very simple. With the potentiometer set up as used for the comparison of E.M.F's., the fall of potential over each resistance is measured. Let the readings on the potentiometer be R' and R'', respectively. Then from the conditions of balance,

$$TI = R'i \qquad \text{and} \qquad SI = R''i,$$

where i denotes the current through the potentiometer resistances.

From this it follows at once that

$$T = S\frac{R'}{R''}$$

and this relation can be determined as accurately as the potentiometer measurements can be made.

92. Calibration of a Voltmeter.—The voltmeter is joined to a storage battery or other source which will maintain the deflection steady at the point it is desired to calibrate. If the voltmeter is direct reading it should read the difference of potential between its own binding posts. To calibrate the scale, then, it is only necessary to measure this same difference of potential by some precise method and compare this measured value with the reading of the voltmeter.

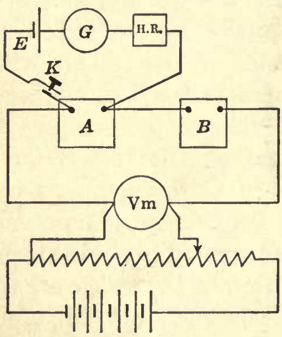


Fig. 48.—Calibration of the voltmeter, *Vm*.

In parallel with the voltmeter is joined a circuit consisting of two resistance boxes, *A* and *B*, and in parallel with *A* is a circuit containing the galvanometer, a standard cell, a high resistance and a key. It is best to have about a thousand ohms in *A* and *B* together for each volt read by the voltmeter. A preliminary calculation will give the resistance which should be placed in each *A* and *B* to give a fall of potential over *A* about equal to the E.M.F. of the standard cell. When these approximate resistances have been placed in *A* and *B*, the key, *K*, can be quickly and cautiously tapped and the direction of the deflection noted. In using a standard cell in this way as little current as possible should be allowed to flow through it. Even a slight polarization will lower its E.M.F. by an unknown amount and then it can no longer be called a "standard" cell. Furthermore, nothing is gained by deflecting the galvanometer "off the scale," as a deflection of a centimeter or two takes less

time and is fully as definite as the larger deflection. The high resistance is inserted for this very protection of the cell and therefore is better than a shunt on the galvanometer.

By adjusting the values of A and B the galvanometer deflection can be reduced to zero. The high resistance can be short circuited for the final balance if the deflections are small. When thus balanced, Kirchhoff's law gives for the circuit through the voltmeter and A and B,

$$Ai + Bi - SI = 0,$$

where SI is the fall of potential over the voltmeter and is what should be indicated on its scale.

Similarly for the circuit through the standard cell

$$Ai = E$$

Eliminating i by division,

$$SI = E\frac{A + B}{A}$$

If V is the reading of the voltmeter, then

$$\mathrm{V} + c = SI,$$

and the correction to be applied at this point is,

$$c = SI - V$$

The computations can be greatly reduced if A is kept at the value $A = 1000\ E$. Thus if the E.M.F. of the standard cell is 1.018 volts at the temperature of the room, A would then be set at 1018 ohms and all of the adjustment made by changing B. The values of SI are then given directly by the sum of A and B divided by 1000.

Other points on the scale can be obtained by using a different number of cells in the main battery, or, if the voltmeter is low reading, by adding some resistance in series with the battery. If the battery cannot be divided into a sufficient number of steps a useful method is to join across the battery terminals a high resistance rheostat and connect the voltmeter

to one terminal and to the sliding contact. Any desired voltage can then be obtained up to the maximum of the battery by simply sliding the movable contact along the rheostat. It is evident that this method can not be used to calibrate points below the E.M.F. of the standard cell.

A calibration curve should be plotted, with voltmeter readings as abscissæ and the corresponding corrections as ordinates. If there is a "zero correction" because the needle does not indicate zero correctly, such correction should be made before computing the calibration corrections.

93. A More Convenient Method.—In case the voltmeter reading can be varied continuously, or by very small steps, either by means of a sliding rheostat as shown in Fig. 48, or by means of a resistance box in series with the voltmeter as shown in Fig. 49, it will be found more expeditious to set A and B at the values corresponding to a given voltage: and then by varying the rheostat to bring the voltmeter to this voltage, the balance being indicated by the galvanometer the same as before. The voltmeter reading should give this same voltage; if it does not, the discrepancy is the correction which must be applied to the voltmeter reading at this point.

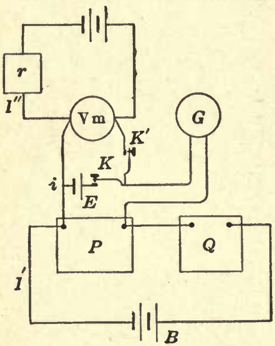


FIG. 49.—Cabralition of the milli-voltmeter, Vm.

94. Calibration of a Low Reading Voltmeter.—A low reading voltmeter can be readily calibrated by the potentiometer method described in Article 86 above. The voltmeter is connected to a battery of one or two good dry cells with a resistance box, r, in series. By varying the amount of resistance in r, the pointer of the voltmeter can be brought to any desired part of the scale. The voltmeter, while still connected with its battery, is also inserted in the galvanometer circuit of the potentiometer in place of the standard cell. There

will thus be two electric circuits, each with its own battery; one battery supplying the current through r and the voltmeter, while the potentiometer current through P and Q comes from the battery B. The galvanometer circuit connects these two circuits, and the current through it may be in either direction, or made zero by adjusting the resistance in P.

Applying Kirchhoff's second law to the galvanometer circuit gives

$$S(I'' + i) + gi - P'(I' - i) = 0,$$

where S is the resistance of the voltmeter. For no current in the galvanometer,

$$SI'' = P'I'$$

The value of the constant current I' is determined by using a cell of known E.M.F. in the same manner as explained in Article 86 (eq. 5). The voltmeter, with its battery, is removed from the galvanometer circuit by leaving K' open, and the standard cell is inserted in the same circuit by using the key K as shown in Fig. 49. The resistance in P is now adjusted to a new value P'' to give no current through the galvanometer. Then,

$$E = P''I'$$

since by keeping $P + Q$ constant the current is the same as before. Therefore,

$$SI'' = E\frac{P'}{P''},$$

and this is what the voltmeter should read. If the voltmeter reading is V, the correction to be applied is

$$c = E\frac{P'}{P''} - V$$

The correction curve should be drawn with the observed voltmeter readings for abscissæ and the corresponding corrections for ordinates.

95. Measurement of Current by Means of a Standard Cell. —A current can be measured quickly and approximately by

reading an ammeter, through which the current is made to pass. But if it is desired to measure the current more exactly a more refined method is necessary; and the method always used in the laboratory is to pass the current, I, through a coil whose resistance, R, is accurately known, and measure the resulting fall of potential, RI. This can be done by any method for measuring potential differences that will give the required degree of accuracy.

In the methods here described the measurements are expressed in terms of a standard cell either by the regular potentiometer method described in Article 86, or by direct comparison like the method for calibrating a voltmeter.

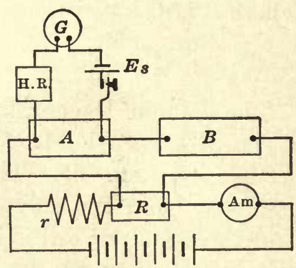


FIG. 50.—Calibration of the ammeter, *Am.*

96. Calibration of an Ammeter.—For the calibration of an ammeter the instrument is joined in series with a standard resistance. The same current must therefore pass through them both. Its amount is determined by measuring the fall of potential over the standard resistance and this value is compared with the reading of the ammeter. The difference is the ammeter correction. In the figure the current from the battery flows in series through the ammeter, the standard resistance R, and an adjustable resistance, r. In parallel with R is a circuit of much higher resistance and consisting of two well adjusted resistance boxes. In parallel with one of these is the galvanometer with a standard cell and a high resistance of about 10,000 ohms to prevent too great a current through the standard cell.

Let the currents through the ammeter, standard resistance, B, and the galvanometer be denoted by I, I', i and i' respectively. Applying Kirchhoff's law to the circuit containing A and the galvanometer, gives, for the case of balance,

$$E_s = Ai.$$

At the same instant the circuit ABR gives

$$RI' = (A + B)\, i$$

Solving for the current in R

$$I' = \frac{E_s}{R} \quad \frac{A + B}{A}$$

Since $I = I' + i$, we have finally,

$$I = \frac{E_s}{R} \quad \frac{A + B + R}{A}$$

It is evident that if A is kept at some round number, say 5000, and the adjustment made by varying B, the computations will be much simplified.

If the reading of the ammeter is I_A then

$$I_A + c = I,$$

and the correction to be added to this reading is

$$c = I - I_A$$

Calibration of Ammeter No. ....

Am. Zero = E_s = at °C. R =

Ammeter		Standard cell		Resistances			Correction
Reading	Corrected	Temp.	E.M.F.	A	B	I	

97. Calibration of a Mil-ammeter. *Potentiometer Method.*—In this method small currents can be measured as well as large ones, and if a potentiometer can be obtained, it is a more convenient method than the preceding. For the best results the known low resistance, C, should be a standard manganin resistance provided with permanent potential terminals.

The ammeter is connected in series with a storage battery, a variable rheostat, and the standard resistance. The fall of potential over the latter is measured by the potentiometer, PQ, in the usual way. The best results are obtained when this is about one volt By means of the double throw switch, S, either C or the standard cell, E, can be thrown into the galvanometer circuit. In the latter case, and after a balance has been obtained by adjusting P until there is no deflection of the galvanometer,

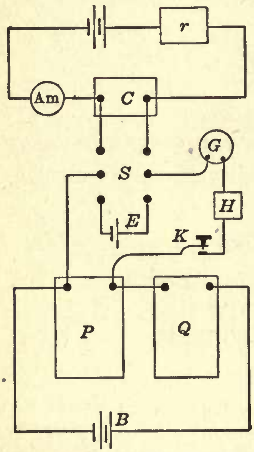


FIG. 51.—Calibration of the ammeter, *Am.*

$$E = P'i.$$

When the coil C, carrying a current I, has been substituted for the standard cell and a balance obtained by readjusting P to some new value, P'', we have,

$$CI = P''i.$$

From which,

$$I = \frac{E}{C}\ \frac{P''}{P'}$$

This value for the current is compared with the ammeter reading, I_A, and the corresponding correction is

$$c = I - I_A.$$

A calibration curve should be drawn, using the observed ammeter readings as abscissæ and the corresponding corrections for ordinates.

CHAPTER VIII

MEASUREMENT OF POWER

104 and 102 A

98. The measurement of electrical power usually resolves itself into the simultaneous measurement of E.M.F. and current. As stated in Article 11, the unit of power is the Watt, and is the power expended by a current of one ampere under a potential difference of one volt. In Article 21, there was given a simple method for measuring the power expended in a circuit by the current from a battery, using an ammeter and a voltmeter. A single instrument combining in itself the functions of both an ammeter and a voltmeter is called a wattmeter. With such an instrument the power may be read directly from a single scale, in the same way as the current is read from the scale of an ammeter.

99. The Use of an Electrodynamometer for the Measurement of Power.—An ammeter and a voltmeter connected as shown in Fig. 52 (a) for the measurement of a resistance will give at the same time the power expended in R. Let B denote the source of the current. The voltmeter, Vm, measures the fall of potential, E, between the terminals, while the ammeter, Am, gives the value of the current. The product, $EI = W$, gives the power in watts.

This result can be expressed in a different form. If in place of a direct reading voltmeter there had been a large resistance of S ohms in series with a mil-ammeter for measuring the current, i, through it, then

$$E = Si, \quad \text{and} \quad W = SiI.$$

In this form it is seen that the measurement of power implies the product of two currents; and in Article 78 it was

seen that an electrodynamometer is an instrument for measuring the product of two currents. Therefore an electrodynamometer can be used as a wattmeter if it is connected into the circuit in the proper manner for this purpose.

Let R, Fig. 52 (c), be the circuit in which the power is to be measured. The low resistance coil, a, of the wattmeter W is connected in series with R as was the ammeter of Fig. 52 (a). The other coil, v, is joined in series with a resistance of several hundred ohms to form a shunt circuit of high resistance, and this is connected in the place of the voltmeter to measure the fall of potential over R and a. For let i denote the value of the current through this shunt circuit, and S its resistance.

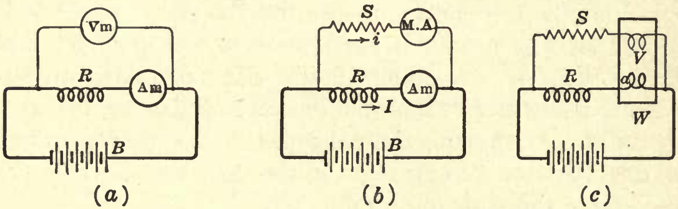


FIG. 52.—Measurement of power.

The fall of potential is then Si, as in Fig. 52 (b). This current i through one coil of the instrument, together with the main current I through the other coil, will produce a deflection D, proportional to the product of the two currents. From the equation of the electrodynamometer,

$$iI = A^2D,$$

where A is the same constant that was previously determined.

Since the power being expended in R is $W = SiI$, we now have,

$$W = SiI = SA^2D.$$

If the constant of the instrument is known, then SA^2 becomes the factor for reducing the scale readings to watts.

In case this factor is unity, as it can be made by adjusting the value of S, the wattmeter is said to be direct reading.

It may be that the value of A is not known, but instead there is a calibration curve for the instrument when used as an electrodynamometer. In this case the value of A^2D can be obtained directly from the curve, for it is the square of the current I' which would produce the same deflection, D.

Thus the power expended in R is

$$W = SI'^2,$$

where S is the resistance of the shunt circuit and I' is not any real current, but it is the current which gave the same deflection when the instrument was used as an electrodynamometer and the value of which can be obtained from the calibration curve.

100. The Weston Wattmeter.—The Weston wattmeter consists, essentially, of a moving coil electrodynamometer. The fixed coil is wound in two sections on a long cylindrical tube, within and between which is the movable coil. The latter is wound with fine wire upon a very short section of a cylindrical tube of somewhat smaller diameter than the fixed coil, and supported on pivots so that it can readily turn about its vertical diameter. Attached to the movable coil is a long light pointer which moves over a graduated scale.

In the position of rest the axis of the movable coil makes an angle of about 45° with the axis of the fixed coil. When deflected so that the pointer is at the middle of the scale the two coils are at right angles. At the extreme end of the scale the coil stands at 45° on the other side of the symmetrical position. This gives a fairly uniform scale over its entire length. A spiral spring brings the coil to the zero position and provides the torque necessary to balance the electrodynamic couple due to the currents in the two coils.

In addition to the main fixed coil there is another fixed coil of fine wire and having the same number of turns as the other, so that a current sent through one coil and back through

the other will produce no magnetic field at the place of the movable coil. It is then possible to compensate for the effect of the shunt current passing through the series coil, for the shunt current can be lead back through this second coil and thus be made to neutralize its action upon the movable coil. When it is desired not to use the compensation coil it is replaced by an equal resistance, this connection being brought out to a third binding post.

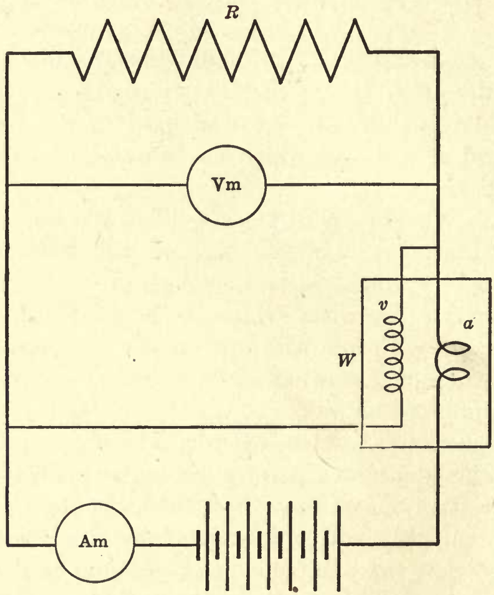


FIG. 53.

In the wattmeter reading up to 150 watts the resistance of the series coil is 0.3 ohm, and that of the shunt circuit is 2600 ohms. The compensation winding is about 3 ohms.

101. Comparison of a Wattmeter with an Ammeter and a Voltmeter.—The reading of a wattmeter can be compared with the power measured by an ammeter and a voltmeter, provided that the latter instruments are connected to measure precisely

the same power as the wattmeter. This means, for the uncompensated wattmeter, that the current through the series coil of the wattmeter must be measured by the ammeter, and the voltmeter must be connected so as to measure the same fall of potential as the shunt coil of the wattmeter.

This is accomplished by the connections shown in Fig. 53. The power thus measured is not that expended in R alone, but it includes the power expended in the voltmeter and in the shunt circuit of the wattmeter. But since both instruments measure this same power, the reading of the wattmeter should agree with the product of the readings from the ammeter and voltmeter.

If the wattmeter is compensated so that the power measured by it does not include the power expended in its own shunt circuit, then the ammeter must be connected so as not to measure this shunt current. But as it is not possible to connect the ammeter and voltmeter so as not to measure the power expended in one or the other of them, the best arrangement will be to join the voltmeter in parallel with R as shown in Fig. 54. The power measured by the ammeter and voltmeter will be that expended in both R and the voltmeter; that measured by the wattmeter will be greater than this by the amount of power expended in the ammeter. The latter can be computed from the formula rI^2, and added to the product of the readings from the ammeter and voltmeter. With this slight correction the wattmeter reading should equal VI.

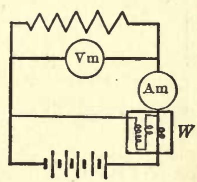


FIG. 54.

102. Power Expended in a Rheostat.—*a. When carrying a constant current.* The object of this exercise is to give the student some personal experience in the measurement of power and in the careful use of a variable rheostat. In this first part a variable resistance is joined in series with a large E.M.F. and considerable other resistance, so that the variations in r will not materially change the value of the current through the

circuit. If desired this variable resistance may include an ammeter, and the current may be kept always at the same value.

The wattmeter is connected to r as shown in the figure, and readings of the power expended in the rheostat are taken for the entire range of the resistance. If the values of the latter are not known they can be measured by one of the methods previously given. A curve should be drawn, using the resistances in the rheostat as abscissæ and the corresponding amounts of power as ordinates. The report should contain a discussion explaining why this curve comes out with the form it has.

b. When under constant voltage. In this part of the exercise the arrangement is much the same as before, except that the high E.M.F. is replaced by a few cells of a storage battery, and r is now the only resistance in the circuit. Starting with the largest values of r, readings of the wattmeter are taken and plotted as before. It will not be safe to reduce r to zero, and readings should be continued only for current values that are not too large for the apparatus used. This portion of the report should give a discussion of what would probably happen if the rheostat were reduced to zero.

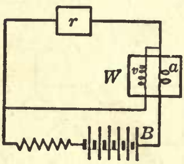


FIG. 55.

102a. Efficiency of Electric Lamps.—An interesting exercise, and one that furnishes considerable information, is the determination of the efficiency of various types of electric lamps. By means of a wattmeter the amount of electrical energy supplied to the lamp is readily measured. The light that the lamp furnishes can be measured by a photometer. The efficiency of the lamp is expressed as the ratio of the candle power of the lamp to the electrical power expended, and is given as so many "candles per watt." Thus the efficiency is not an abstract ratio, as in most cases, because it is not possible to measure light in watts. But this does not prevent a satisfactory comparison of different lamps.

Any good form of photometer can be used to measure the candle power of the lamp being examined. If none is at hand, a simple form can be made by standing some object in front of a white screen. The standard light and the one being measured will each cast a shadow of the object on the screen, and by varying the distance of one of the lights from the screen the intensities of the two shadows can be made equal. The intensities of the two lights are then to each other as the squares of their respective distances from the screen.

If a number of different kinds of lamps are available it is instructive to measure the efficiency of each one. Tungsten lamps can be compared with carbon lamps, and lamps that have been in use for a long time can be compared with new lamps of the same kind. It is also interesting to determine the efficiency of the same lamp when burned at different voltages, and it is well to plot a curve with efficiencies as ordinates with the corresponding voltages for abscissae. Incidentally the curve showing the variation of candle power with voltage can be plotted.

103. Measurement of Power in Terms of a Standard Cell.— In the preceding chapters there have been given methods for measuring either a current or a difference of potential in terms of the E.M.F. of a standard cell. By combining two of these methods it is possible to measure power in like manner, and this is especially useful when it is desired to know accurately the value of a given amount of power. For example, in some methods of calorimetry it is necessary to have supplied a known amount of heat. Often the actual amount is not essential, but whatever it is it must be known to a high degree of accuracy. In such cases it is convenient to generate the heat by an electric current flowing through a resistance coil, and then to measure the electrical power expended in terms of a standard cell of known E.M.F. This means the measurement of both the current and the fall of potential, as the resistance of the coil usually can not be accurately determined under the conditions of actual use.

One convenient arrangement, which is capable of wide variation in the amount of power that can be measured, is shown in the figure. The heating coil in which the power is expended is denoted by H. In series with this is a coil C, of sufficient current carrying capacity not to be heated by the current through it. There is also a variable resistance, r, by which the current can be brought to any desired value. The current through C is measured by the method for calibrating an ammeter (see Article 96) and the fall of potential over H is measured by the method for calibrating a voltmeter (see Article 92).

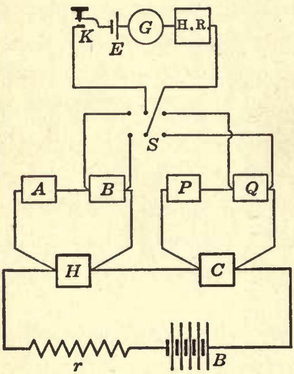


Fig. 56.—Measurement of power expended in H in terms of a standard cell, E.

The standard cell is joined in series with a sensitive galvanometer and a high resistance. The circuit thus formed is connected to the middle of a double throw switch S. One end of this switch is connected to B, which is a portion of a shunt around H. By adjusting A and B, the fall of potential over the latter can be made equal to the E.M.F. of the standard cell, as shown by no deflection of the galvanometer when the key K is closed. The total fall of potential over H is, then,

$$V = \frac{A + B}{B} E_s$$

In the same way there is a shunt circuit, PQ, in parallel with C, and the part Q is joined to the other end of the double throw switch. When the switch is thrown to this side, P and Q can be adjusted to give no deflection of the galvanometer when the key is closed. This means that the fall of potential over Q has been made the same as the E.M.F. of the standard cell, and the total fall over C is computed as shown above

for H. This, divided by the resistance C, gives the current through C as

$$I = \frac{E_s}{C}\,\frac{P+Q}{Q}$$

A little consideration will show that the main current through the battery is larger than I by the amount of current that flows through the shunt circuit PQ; and the current through H is smaller than the main current by the amount of current that flows through the shunt AB. If Q is set equal to B, then since the fall of potential over each is the same, the currents through these two shunts will be equal. Therefore the current through H will be I, the current through C. The power expended in H is, then,

$$W = VI = \frac{E^2}{B^2}\,\frac{(A+B)(P+Q)}{C},$$

when the two shunt currents have thus been made equal.

104. Calibration of a Non-compensated Wattmeter.—The wattmeter in this case may be an electrodynamometer with two separate coils, or it may be the regular Weston wattmeter used without the compensation coil. The series coil of the wattmeter is connected in series with a resistance, H, in which can be expended the power measured by the wattmeter. There is also in series a standard resistance, C, whose value is accurately known and which has sufficient current carrying capacity not to be heated by the currents used in the calibration. A variable resistance, r, and a storage battery, z, complete the main circuit. The shunt coil, ab, of the wattmeter is connected in parallel with H. Two well-known resistances, A and B, are also connected in parallel with H. The power measured by the wattmeter is then the total power expended in H, the wattmeter shunt circuit, and the circuit consisting of A and B. This power is the product of the current through these three in parallel and the potential difference between m and n. Both of these quantities are determined by the potentiometer.

The potentiometer is represented by the three resistances, P, Q, and R. Across P is joined the galvanometer and standard cell in the usual way. When P has been adjusted for a balance,

$$E = iP, \qquad \text{or} \qquad i = \frac{E}{P},$$

where E is the E.M.F. of the standard cell and i is the steady constant current that is flowing through the main circuit of the potentiometer.

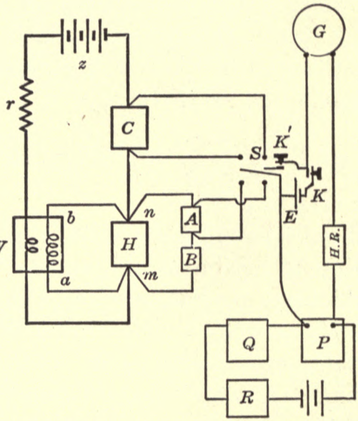


FIG. 57.—Calibration of the wattmeter, W.

To determine the value of the current through the standard resistance, wires are brought from the terminals of C to the double throw switch S. When this switch is thrown up and K' is closed, C is connected into the galvanometer circuit in the place of the standard cell. Readjusting the potentiometer for a new balance, P', gives,

$$IC = iP' = \frac{EP'}{P}$$

and hence,

$$I = \frac{EP'}{CP}.$$

To determine the fall of potential over H the double throw switch is thrown down, thus connecting the resistance A into the galvanometer circuit. Let i' denote the value of the small shunt current flowing through A and B. Then adjusting the potentiometer for a balance,

$$i'A = iP'' = \frac{EP''}{P},$$

where P'' is the new reading of the potentiometer. The fall of potential over both A and B is $\frac{A + B}{A}$ times as large, or,

$$V = \frac{EP''}{P} \frac{A + B}{A}$$

and this is the same as the fall of potential over H.

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2}{P^2} \frac{P'P''}{C} \frac{A + B}{A}$$

If the reading of the wattmeter is W, the correction to be added to this reading is,

$$c = W' - W.$$

Different readings of the wattmeter are secured by changing the current through H. A calibration curve can be plotted, using the readings of the wattmeter as abscissæ and the corresponding corrections for ordinates.

105. Calibration of a Compensated Wattmeter.—For this purpose a potentiometer may be used, as in the preceding method, but as this instrument is not always at hand a method using resistance boxes is given here. The principal difference between the compensated wattmeter and the uncompensated one is that the former measures only the power expended in the circuit to which it is attached, while the latter measures, in addition to this, the power expended in its own shunt circuit.

Thus let W, Fig. 58, represent the wattmeter connected

in the circuit to measure the power expended in H and C together. C is a standard resistance for use in the measurement of the current, and H is sufficient other resistance to give the required amount of power. As the power expended in the shunt coil, ab, is not measured by the wattmeter, it should

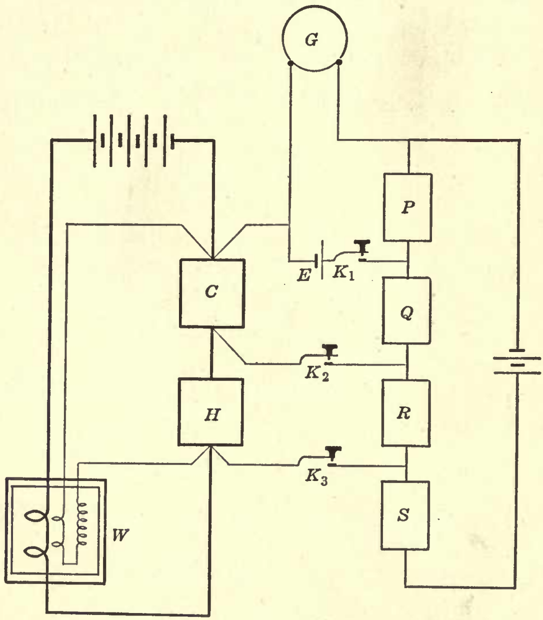


FIG. 58.—Calibration of the Wattmeter, W.

not be measured by the standard cell; therefore C is placed inside next to H.

The calibration circuit consists of four resistance boxes, P, Q, R, and S, joined in series with a battery of a few volts sufficient to maintain a small constant current through the circuit. This circuit is joined through the galvanometer to the standard cell and to the wattmeter circuit in three places as shown, each connection being provided with a key. When

finally balanced no current flows through any of these connections.

The measurements are made as follows. First P is set at some convenient value, say 1000 ohms, and some of the remaining resistances are then changed until there is no deflection of the galvanometer upon closing the key K_1. This fixes the total resistance of this circuit and it is thereafter kept constant at this amount. The fall of potential over C should be a little larger than the E.M.F., of the standard cell. It will then require a little more resistance added to P to give no deflection when the key K_2 is used. This is added by varying Q and S, keeping their sum constant, until there is no deflection of the galvanometer upon closing the key K_2. This balance measures the value of the current through C, for

$$IC = i(P + Q) = (P + Q)\frac{E}{P},$$

and therefore,

$$I = \frac{E}{C}\frac{P + Q}{P}.$$

This is the effective current actuating the wattmeter.

The fall of potential over CH is measured in the same way. This will be greater than that over C alone, therefore for a balance it will require a resistance greater than $P + Q$. The resistance R is now varied, keeping $R + S$ constant, until there is no deflection of the galvanometer upon closing K_3. The fall of potential over CH is then the same as that over the three resistances, P, Q, and R. That is,

$$V = i(P + Q + R) = (P + Q + R)\frac{E}{P}.$$

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2(P + Q)(P + Q + R)}{P^2C}$$

If the reading of the wattmeter is W, the correction to be added to this reading is,

$$c = W' - W.$$

In case V is too large to be measured directly as here shown, there may be placed a shunt, AB, around H as shown in the preceding method and the potential fall over A alone measured. The total fall of potential over H is then readily computed. The addition of this shunt will make no difference in the wattmeter, since H with its shunt now replaces H alone and the wattmeter measures whatever power is expended in either arrangement.

106. Calibration of a High Reading Wattmeter.—A high reading wattmeter is one that measures large amounts of power. In calibrating such a wattmeter it is often impossible, and usually it is inconvenient, to expend sufficient power to bring the reading up to the high values indicated on the scale. But this is not necessary, for all that is required is that there shall be a large current through the series coil and a small current at high voltage through the shunt circuit. By using different batteries to supply these two currents there need be no great expenditure of energy. As shown in the figure, the battery B, consisting of one or two large storage battery cells, supplies a large current through the series coil of the wattmeter and the low standard resistance C. The latter should be of such a value that the fall of potential over it will be a volt or less in order that this may be readily measured by the potentiometer. Since the resistance of the shunt circuit is large it will require a large number of cells in the battery B', but these cells can be small as only a small current will be needed. In parallel with this circuit is placed another high resistance circuit, AB, so divided that the fall of potential over the portion A shall be about one volt. The calibration is then the same as given above for the case of an uncompensated wattmeter, and

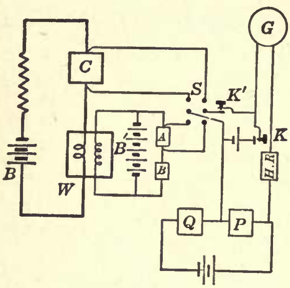


FIG. 59.—Calibration of the Wattmeter, W.

the wattmeter should read the product of the current through C and the voltage across A and B.

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = V\,I - W.$$

In making this calibration it is best to have the currents through the wattmeter about the same as will be used when the instrument is measuring power.

CHAPTER IX

MEASUREMENT OF CAPACITY

107. Laws of Condensers.—When two or more condensers are joined to the same E.M.F. each one, of course, becomes charged to this difference of potential. The total charge is

$$Q = Q_1 + Q_2 = C_1E + C_2E = CE,$$

where Q_1 and Q_2 are the charges in each condenser. This combination acts, then, like a single condenser whose capacity is

$$C = C_1 + C_2$$

Hence,

Law I.—The combined capacity of several condensers in parallel is equal to the sum of the separate capacities.

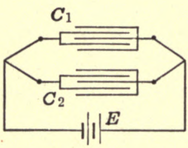


FIG. 60.—Condensers in parallel.

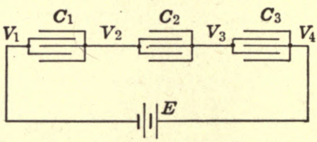


FIG. 61.—Condensers in series.

If the condensers are joined in series as shown in Fig. 61 it is evident that the difference of potential over each condenser is only a part of the total E.M.F. of the battery. The amount of charge in each of the condensers will be the same, for, being joined in series, whatever electricity leaves one must go into the other, if the intermediate parts are well insulated.

The potential differences across each condenser, respectively will be,

$$V_1 - V_2 = \frac{Q}{C_1}$$

$$V_2 - V_3 = \frac{Q}{C_2}$$

$$V_3 - V_4 = \frac{Q}{C_3}$$

Adding these equations gives

$$V_1 - V_4 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q}{C}$$

Hence the equivalent capacity, C, of the combination is given by the relation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Law II.—When several condensers are connected in series, the joint capacity is the reciprocal of the sum of the reciprocals of the several capacities.

This is similar to the law for resistances in parallel.

108. Comparison of Capacities by Direct Deflection.—When a condenser of capacity C farads is charged to a potential difference of E volts, the quantity it contains is

$$Q = CE, \text{ coulombs.} \qquad (1)$$

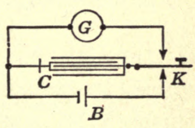


FIG. 62.

If this quantity is discharged through a ballistic galvanometer, giving a fling of d mm. as measured on the scale, we have

$$Q = kd. \qquad (2)$$

Combining (1) and (2) gives

$$C = \frac{k}{E}d \qquad (3)$$

Now if the same experiment is repeated using the same battery and galvanometer, but with another condenser of capacity C_1 we have

$$C_1 = \frac{k}{E} d_1 \tag{4}$$

and from (3)

$$C_1 = C \frac{d_1}{d} \tag{5}$$

If C is a known capacity then the value of C_1 can be determined as exactly as the flings d and d_1 can be measured. Each of these should be taken several times and the mean values used in the computation.

109. Bridge Method for Comparing Two Capacities.—This is a null method and therefore capable of more exact measurements than the preceding. The two condensers are placed in two arms of a Wheatstone bridge setup as shown in Fig. 63.

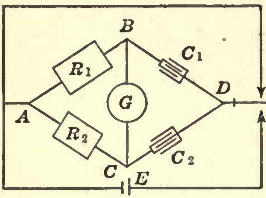


FIG. 63.—Bridge method.

When the key is depressed both condensers become charged to the full potential difference of E, and the points A, B and C all come to the same potential. If no charge passes through the galvanometer then C_1 is charged through R_1 and C_2 through R_2. During the very short interval that is required to charge the condensers there will be transient currents through R_1 and R_2, and perhaps in the galvanometer also.

By working the charge and discharge key quickly the deflection of the galvanometer may be checked even when R_1 and R_2 are far from a balance. The galvanometer should be thus protected.

Applying Kirchhoff's second law to the circuit $B\ A\ C\ B$, at any instant while the condensers are being charged gives,

$$R_1 i_1 + L_1 \frac{di_1}{dt} - R_2 i_2 - L_2 \frac{di_2}{dt} + Gi + L \frac{di}{dt} = 0$$

where $L_1 \frac{di_1}{dt}$, etc., are the E.M.F's. due to self induction in the corresponding branches.

Multiplying this equation by dt and integrating it from $t = 0$, to $t = T$, for the time limits, and between the corresponding limits $i = 0$ and $i = 0$ for the currents gives

$$R_1q_1 - R_2q_2 + Gq = 0.$$

It is to be noticed that no deflection of the galvanometer does not mean no current through it, as there may be currents through it in both directions before the condensers are fully charged. But zero deflection means that as much electricity has passed through the galvanometer in one direction as in the other, and that, taking account of signs, the total quantity through the galvanometer has been zero. Hence, when R_1 and R_2 are adjusted so that opening or closing K produces no deflection of the galvanometer, $q = 0$.
Then

$$R_1q_1 = R_2q_2 \qquad \text{(A)}$$

But since no charge passed through the galvanometer,

$$q_1 = C_1E \qquad \text{and} \qquad q_2 = C_2E,$$

Hence, substituting in (A), gives

$$C_1 = C_2\frac{R_2}{R_1}.$$

The resistances R_1 and R_2 should be large, 5000 ohms or more, so that the fall of potential produced by the small charging currents may be appreciable. While self inductance in these arms does not affect the result, as seen by the result of the preceding integration, a large amount may render it more difficult to determine when a balance has been reached.

Make five determinations of each unknown capacity by using various values for R_1 and finding the corresponding values of R_2. Check results by measuring the capacity of the condensers when joined in series and in parallel.

The data can be recorded in a form like the following:

To Measure the Capacity of . . . .

R_1	R_2	C_2	C_1

110. Comparison of Capacities by Gott's Method.—This is another bridge method and differs in arrangement from the preceding only by having the galvanometer and battery interchanged. In the bridge method the balance is obtained for the conditions which exist *during* the charging, or the discharging, of the condensers. In the present method the capacities of the condensers are compared after everything has reached the steady and permanent condition.

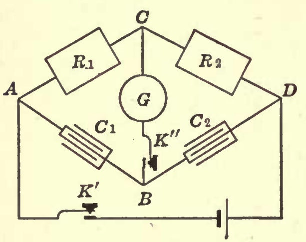


Fig. 64.—Gott's method.

The arrangement is shown in Fig. 64. When the battery key, K', is closed the two condensers in series are charged to the difference of potential between A and D. The point B, between the two condensers, has a potential intermediate between that of A and D. The measurement consists in adjusting R_1 and R_2 until C on the upper circuit, has the same potential as B in precisely the same way as in the measurement of resistance by the Wheatstone bridge method. When this adjustment is correct, there will be no deflection of the galvanometer upon closing K''.

Since the condensers are joined in series, each one must contain the same charge. The point B is insulated as long as K'' remains open, and therefore whatever charge appears on one condenser must have come from the other one. Moreover, for a balance the closing of K'' produces no deflection of the

galvanometer, i.e., there is no change in the charges on the conductors joined at B.

Applying Kirchhoff's second law to the circuits $BACB$ and $DBCD$ gives,

$$\frac{q}{C_1} - R_1 i = 0 \qquad \text{and} \qquad \frac{q}{C_2} - R_2 i = 0$$

from which,

$$C_1 = C_2 \frac{R_2}{R_1}.$$

It should be noted that closing K'' brings B to the potential of C whether there is a balance or not. A second closing of K'' can produce no deflection. It is necessary therefore to completely discharge the condensers after each closing of K''. This can be most quickly done by opening K' before opening K''.

If one of the condensers has considerable absorption or leakage it will seriously influence the results, for after some of the charge has leaked away it is no longer true to say that the charges in the two condensers are equal. The effect of this source of error is reduced by closing K'' as soon as possible after closing K'. Sometimes a rapidly alternating E.M.F. is used in place of the battery and key, and a telephone receiver instead of the galvanometer and key. This would only be allowable when the resistances R_1 and R_2 are free from self induction and capacity.

If this is true, it is also allowable to omit K'' from Fig. 64 and obsrve the deflections of the galvanometer when K' is closed or opened.

The data can be recorded in the same form as used for the preceding experiment.

111. Comparison of Capacities by the Method of Mixtures. —This method was devised by Lord Kelvin to avoid some of the difficulties in the preceding methods. It is especially applicable to cases where the two capacities are very dissimilar, e.g., if the capacity of a long cable is to be compared with that

of a standard condenser. The method consists of charging the condensers to such potentials that each will contain the same quantity. They are then discharged, the one into the other, and the charges allowed to mix. If the charges are not equal, the difference will remain in the condensers and is later discharged through the galvanometer.

The arrangements and connections are shown in Fig. 65. Two moderately high resistance boxes, R_1 and R_2 are joined in series with a battery of a few cells. In parallel with each

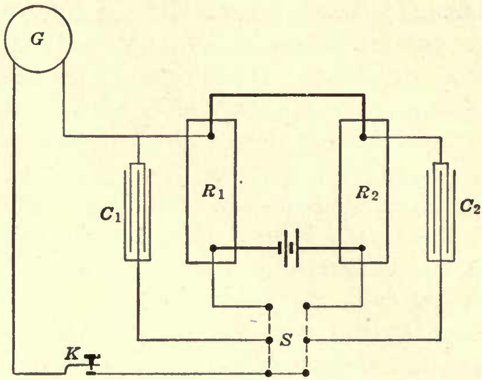


FIG. 65.—Method of mixtures.

resistance is joined one of the capacities to be compared, the connection being made through the double switch, S. Each condenser is thus charged to the potential difference over the corresponding resistance. The charges are, then,

$$Q_1 = C_1R_1i \quad \text{and} \quad Q_2 = C_2R_2i,$$

where i denotes the steady current through R_1 and R_2.

By throwing the switch the other way the condensers are joined together and the charges mix, the difference, if any, being discharged through the galvanometer by the key, K. No deflection indicates that the charges are equal, and

R_1 and R_1 are adjusted until this balance is obtained. Then,

$$Q_1 = Q_2 \quad \text{or} \quad C_1R_1i = C_2R_2i,$$

from which

$$C_1 = C_2\frac{R_2}{R_1}$$

In order to avoid as far as possible the effects of leakage and absorption the keys should be worked as quickly and uniformly as possible. This can best be done by using the special testing key, shown in principle in Fig. 66. This is a regular Wheatstone bridge key with the addition of a break contact on the top of each blade. The top and bottom blades of the key are both joined to one condenser, say C_1, and the middle blade is joined to the other condenser, C_2, as indicated in the figure. The condensers are thus connected to the resistances through the contacts R_1 and R_2 respectively. The key is provided with an extra insulation block to prevent the upper blades from coming in contact when the key is depressed. Also the usual insulation between the lower blades can be moved to one side, thus allowing these blades to come in contact.

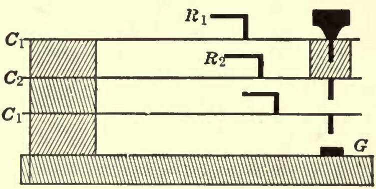


FIG. 66.—Triple make-and-break key.

When the key is depressed, the first action is to disconnect the condensers C_1 and C_2 from the charging contacts, R_1 and R_2. When the key is further depressed the two lower blades come in contact, thus joining the condensers together and mixing their charges. Finally, when the lower blade touches the last contact, G, the charge remaining in the condensers after the mixing, is discharged through the galvanometer. All of the operations for this experiment are thus performed while the key is being depressed.

The data form in Article 109 may be used here also.

112. Study of Residual Discharges.—When a condenser or a cable is charged for a long time and then discharged it is nearly always found that the quantity of electricity obtained from the condenser on discharge is less than the total amount of the original charge. The remainder of the original charge is said to be "absorbed," meaning thereby that this charge remains in the condenser after the plates have been brought to the same potential, but not specifying the manner in which it is retained. After a short interval of time a portion of this absorbed charge is released and can be discharged by again joining the two plates of the condenser; and this is termed a residual discharge. Several such residual discharges can be obtained from ordinary condensers and a great many from a poor condenser.

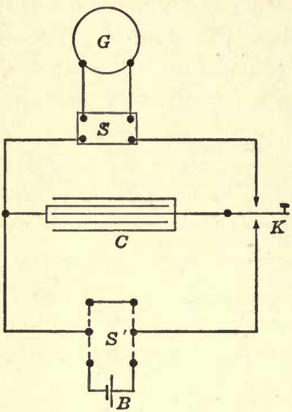


FIG. 67.

This phenomenon depends upon the material composing the dielectric between the plates of the condenser, and is the more marked the greater the heterogeneity of the dielectric. In case the dielectric is air, or a sheet cut from a crystal of quartz or iceland spar (i.e., a homogeneous substance) there is no residual discharge. The charge which is thus absorbed and can be recovered later should not be confused with any leakage there may be through the condenser.

To make a short study of residual discharges proceed as follows: Charge a paraffined paper condenser from six or eight cells for 3 minutes. When ready to begin observations on residual discharges disconnect the battery from the condenser and thoroughly discharge the latter by throwing the switch S to the other side for 5 seconds and then leaving it open. Wait 1 minute and again discharge the condenser—this time through the galvanometer and observe the deflection.

The key K is supposed to remain closed on the lower contact shown in Fig. 67. Probably this would be the upper contact on the actual key. Pressing the key to the other contact joint the condenser to the galvanometer. Discharge through the galvanometer at minute intervals for half an hour. Express the results in the form of a curve taking time for abscissæ and for ordinates the sum of all previous discharges. In some instances the sum of the residual discharges is greater than the original discharge. Repeat, charging the condenser for 15 minutes in order to observe the effect of the time of charging upon the amount of the residual charge.

113. The Ballistic Galvanometer.—A ballistic galvanometer is one designed to measure transient currents, or quantities of electricity, like the induced current in a secondary coil or the discharge of a condenser. The duration of the current is very brief as compared with the period of the needle of the galvanometer, and thus the coil remains practically at rest until the entire quantity has passed through it. It is impossible, therefore, to obtain a steady deflection, but the galvanometer coil gives a sudden throw and then settles down to the position of rest. The extent of this throw gives a measure of the quantity of electricity that has passed through the galvanometer: the greater the quantity the greater the throw, the exact relation depending upon the type of galvanometer employed.

These galvanometers are of two general types: (1) The magnet may be fixed, and the coil suspended and movable; or, (2) the magnet may be suspended and movable and the coil fixed.

114. Turning Moment Due to a Current.—In galvanometers of the first type the poles are usually so shaped as to give a radial magnetic field. Then the angular twist of the suspension produced by the couple due to a steady current flowing through the coil, is directly proportional to the current, and is independent of any particular position of the coil. If I is the value of the steady current, and G the torque per unit current

(a constant of the galvanometer), then GI is the moment of the forces due to the steady current I tending to turn the coil. When a condition of equilibrium is reached, we have, by equating moments,

$$GI = a\phi \tag{1}$$

where ϕ is the angle through which the coil has been turned, and a is the torque of the suspension, or the couple exerted by the suspension when the coil is turned through unit angle.

When the current is not steady, but is variable, there can be no steady deflection ϕ, but an angular acceleration is given to the coil, the amount of which is measured by the subsequent fling. If i is the value of the transient current at any instant, then Gi will be the moment of the forces due to this current, and the equation of motion of the coil is,

$$Gi = a\phi + K\frac{d\omega}{dt}$$

But $\phi = 0$, before the suspension is twisted, and therefore at the start,

$$Gi = K\frac{d\omega}{dt} \tag{2}$$

where K is the moment of inertia of the moving system (coil, mirror, etc.), and $\frac{d\omega}{dt}$ the angular acceleration, where ω is the angular velocity.

From (2) we have the relation that,

$$Gidt = Kd\omega \tag{3}$$

Now the integral of idt is simply Q, the total quantity of electricity that has passed through the galvanometer. Therefore, from (3),

$$Q = \frac{K\omega}{G} \tag{4}$$

and it remains to measure ω.

115. To Express ω in Terms of Known Quantities.—The kinetic energy of a system of moment of inertia K and rotating with this angular velocity ω, is

$$K.E. = \frac{1}{2}K\omega^2 \tag{5}$$

and the coil will continue to turn until this energy has all been expended in twisting the suspension. The suspension thus gains potential energy, and when the coil comes to rest (for an instant at the position of its maximum fling), the potential energy of the twisted suspension equals the kinetic energy with which the coil started—save for a slight amount lost through friction, air currents, etc.

The amount of potential energy thus gained by the suspension can be computed independently by considering the work that has been done upon it. The torque exerted by the suspension when twisted an angle ϕ is $a\phi$; and the work to twist it through the further angle $d\phi$ is $a\phi d\phi$. The total work done in twisting the suspension from zero to an angle θ is,

$$\int_0^\theta a\phi \, d\phi = \frac{1}{2} a\theta^2$$

and this is equal to the potential energy gained by the suspension.

Equating the kinetic energy at the beginning of the swing to the potential energy at the end, we have the relation that,

$$\frac{1}{2} K\omega^2 = \frac{1}{2} a\theta^2$$

where θ is the maximum throw.

Solving for ω gives,

$$\omega = \sqrt{\frac{a}{K}}\,\theta,$$

and substituting this value in (4) gives,

$$Q = \frac{K}{G}\sqrt{\frac{a}{K}}\,\theta = \frac{a}{G}\sqrt{\frac{K}{a}}\,\theta.$$

From equation (1) we have the relation that

$$\frac{a}{G} = \frac{I}{\phi}$$

and putting this value into the expression for Q gives,

$$Q = \frac{I}{\phi}\sqrt{\frac{K}{a}}\,\theta. \qquad (6)$$

The value of $\frac{K}{a}$ can be determined by finding the period of oscillation of the moving system.

Usually the deflections, d' and d, as read from the scale, are proportional to the angles ϕ and θ. If they are not they can be corrected by means of a calibration curve. Writing, then, $\frac{d}{d'}$ for $\frac{\theta}{\phi}$, gives

$$Q = \frac{I}{d'}\sqrt{\frac{K}{a}}\, d = F\sqrt{\frac{K}{a}}\, d, \qquad (7)$$

where F is the figure of merit of the galvanometer.[1]

116. Period of Oscillation.—In a galvanometer of the moving coil type the restoring force is entirely due to the suspension, and therefore varies directly as the angle of twist, θ. The equation of motion is then,

$$K\frac{d^2\theta}{dt^2} = -a\theta$$

where θ is the angular displacement, and K and a have the same meanings as above.

We find[2] the solution of this equation to be the simple harmonic function,

$$\theta = c \sin\left[\sqrt{\frac{a}{K}}\,t + f\right],$$

where c and f are the two constants of integration. At the end of the first period the value of θ will return to this same value, but the angle part of the above expression will have increased. This increase at the end of the first period will be 2π, and the angle will now be

$$\sqrt{\frac{a}{K}}\,t + f + 2\pi.$$

If we think of the angle as increasing because the value of t

[1] See Article 40

[2] Murray, Differential Equations, p. 97.

has increased, then at the time $t + T$, corresponding to the end of the first period, the value of the angle will be

$$\sqrt{\frac{a}{K}}(t + T) + f.$$

Since the angle is the same in either case, these two values are equal, and we have

$$\sqrt{\frac{a}{K}}t + f + 2\pi = \sqrt{\frac{a}{K}}(t + T) + f.$$

Solving for T we have

$$T = 2\pi\sqrt{\frac{K}{a}}$$

from which

$$\sqrt{\frac{K}{a}} = \frac{T}{2\pi}$$

Substituting this value in equation (7),

$$Q = \frac{FT}{2\pi}d.$$

Thus our final equation contains only measurable values.

But this equation does not take into account the effect of damping on the reading of the galvanometer. If this is not too great, a correction can be made for it, as follows.

117. Correction for Damping.—Let θ_1, θ_2, θ_3, etc., be the successive deflections of the galvanometer to the right and left when it is allowed to swing freely after the discharge of the condenser through it. It is observed that each deflection is less than the one before it by a certain constant ratio, so that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \text{etc.} = f, \text{ say.}$$

If these deflections are laid off as ordinates, each one being erected at that point on the axis of abscissæ corresponding to the time at which it was observed, they would appear as in

Fig. 68. Time is reckoned from the discharge of the condenser and when the galvanometer coil begins to move. After half of a single period the first throw, θ_1, is observed. The succeeding deflections follow at equal intervals of a whole single period. Knowing the value of f, any given deflection, say θ_1, can be computed from the later readings, since

$$\theta_1 = \theta_2 f = \theta_3 f^2 = \theta_4 f^3 = \text{etc.}$$

where the exponent of f in each case is equal to the number of single periods between the desired and observed deflections.

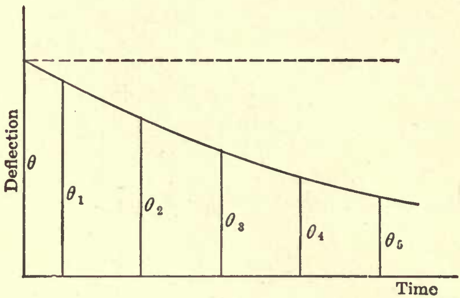


FIG. 68.—Damping of a galvanometer.

Had there been no damping, all of these deflections would have been larger, and each equal to θ, the value of which is

$$\theta = \theta_1 f^{\frac{1}{2}},$$

where the exponent of f is $\frac{1}{2}$ because the interval between θ and θ_1 is half of a single period.

Since

$$f = \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_1 + \theta_2}{\theta_2 + \theta_3} = \frac{S_1}{S_2},$$

the value of f is seen to be also given by the ratio of two consecutive swings—a swing being the full amplitude from the turning point at the right to the turning point at the left.

Using this value of f gives,

$$\theta = \theta_1\left(\frac{S_1}{S_2}\right)^{\frac{1}{2}}.$$

118. Final Formulas.—The final corrected value for Q becomes, then,

$$Q = \frac{FT}{2\pi}\left(\frac{S_1}{S_2}\right)^{\frac{1}{2}} d_1 = kd_1 \tag{8}$$

Where k represents all the constants in this equation, and d_1 is the observed value of the first deflection.

119. Absolute Capacity of a Condenser.—By "absolute capacity" is meant the value of the capacity of a condenser determined independently of any other capacity. The comparison of two condensers can only give their relative capacities.

The fundamental relation for a condenser is

$$Q = CE.$$

If then by means of a ballistic galvanometer we can measure the quantity Q which will charge a condenser to the potential difference E, its capacity can be determined from this relation. The setup is shown in the figure. R is a high resistance of about 100,000 ohms, and together with P and S forms an arrangment for finding the figure of merit of a galvanometer. By closing K'' a steady current can be passed through the galvanometer, and the resulting steady deflection d' observed. If the galvanometer is a sensitive one it will be necessary to use only a few ohms in P while S should be several thousand ohms. The current through the galvanometer is:

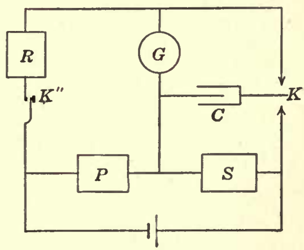


FIG. 69.

$$I = \frac{P}{P+S}\cdot\frac{V}{R+g} = Fd'.$$

The condenser may be joined in as shown where it is charged from the potential across the resistance S. In case this charge is too great the fall of potential across S can be reduced by inserting another resistance in series with the battery. (It is best not to use a shunt with the galvanometer, unless it be for damping.)

The quantity of electricity passing through the galvanometer has been found to be given by the expression,

$$Q = \frac{F\,T}{2\pi} \left(\frac{S_1}{S_2}\right)^{\frac{1}{2}} d_1,$$

where the symbols have the same meanings as before. Each quantity can be determined, and therefore the charge in the condenser can be found.

The difference of potential, E, to which the condenser is charged is,

$$E = \frac{S\,V}{P + S}$$

where V denotes the fall of potential over both P and S. We thus have for the capacity of the condenser,

$$C = \frac{Q}{E} = \frac{T}{2\pi}\,\frac{P}{S(R + g)} \left(\frac{S_1}{S_2}\right)^{\frac{1}{2}} \frac{d_1}{d'}$$

CHAPTER X

THE MAGNETIC CIRCUIT

120. Introduction.—It has been already noticed that an electric current produces a magnetic field, and this phenomenon is especially striking when the current is not steady. In the case of the dynamo and the transformer the current is generated by the varying magnetic field. It is the object of this chapter to inquire more minutely into this intimate relation between the electric current and the magnetic field with which it is associated.

In order to fix ideas by a concrete example let us consider a simple dynamo as outlined in Fig. 70. *A* is the armature which rotates between the pole pieces, *PP*. *FF* are the fields, which are connected above by the yoke, *Y*. The magnetism is produced by an electric current through the many turns of the field coils. The closed path *APFYFPA* is called the magnetic circuit, and in this case it is almost entirely of iron, because iron is one of the best conductors of the magnetic flux.

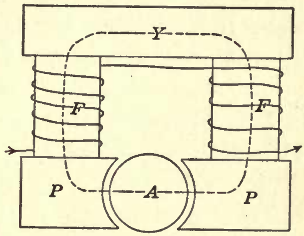


Fig. 70.—A magnetic circuit.

What is this relation between electricity and magnetism? How is the magnetism of the dynamo produced? Why is so much power required to run it? In the pages that follow we will endeavor to find the answers to these questions, and to learn some things regarding the nature of magnetism.

121. The magnetic circuit is analogous to the electric circuit, but with one very important difference. As there are

no insulators for magnetic flux it is not possible to confine the magnetic flux to any chosen path as in the electrical analogy. It is as though the electric circuit were immersed in a conducting bath of salt water where the greater part of the current would follow the best conductor but some portion of it would flow through the surrounding medium. The law of the magnetic circuit is precisely similar to Ohm's law under these conditions, and can be written

$$\text{Magnetic Flux} = \frac{\text{Magneto-motive Force}}{\text{Reluctance}} \qquad (1)$$

This law can be applied to any magnetic circuit, and it holds true for circuits of any material, with one exception. For substances which show magnetic hysteresis—such as iron, steel, nickel, etc.—this law may be applied only when the substance is being magnetized by an *increasing* M.M.F. which is greater than any M.M.F. that has been used since the substance was unmagnetized.

122. Magnetic flux thus has its analogy in electric current. It is always continuous and forms a closed circuit with itself. At one part it may be spread out over a large area and at another part of the circuit be confined within narrow limits, but the total amount of magnetic flux is the same for every cross section of the entire circuit. When once the magnetic flux has been established no further expenditure of energy is required to maintain it. In this respect it is quite different from the electric current.

The direction of the magnetic flux can be represented by "stream lines" in space, these lines being closer together where the flux is more intense and farther apart where the flux spreads out over a wider path. Since the flux is continuous, each line forms a closed curve, and two lines can never intersect or cross one another. There is no limit to the number of such stream lines that may be drawn, but by convention we consider only as many as are numerically equal to the value of the flux as given by equation (1).

The unit of flux is called a *maxwell,* and is represented, then, by one line. These lines are sometimes called "lines of induction." It will be shown later that a magnetic flux is measured by the E.M.F. induced in a conductor which cuts across the flux, and this effect gives a basis for defining the value of a maxwell. Looking forward to Article 139, there is found the following statement.

A maxwell is the amount of magnetic flux cut each second by a conductor in which there is induced an E.M.F. of one C.G.S. unit.

The name "maxwell" for unit magnetic flux was adopted by the International Electrical Congress at Paris in 1900.[1]

If a ballistic galvanometer is connected to the conductor when it is cutting the magnetic flux there will be a deflection of the galvanometer, and this deflection may be taken as a measure of the total flux cut by the conductor. It does not matter whether the conductor is moved, or whether the flux moves across the stationary conductor, the effect on the galvanometer is the same.

If then, for example, it is desired to measure the flux through a bar magnet, a turn of wire is placed around the bar and its ends joined to a ballistic galvanometer. When the magnet is suddenly withdrawn from the loop of wire there is a deflection of the galvanometer which is proportional to the total amount of flux cut across the wire. Since in the first position the loop of wire enclosed all of the flux passing through the magnet, and after withdrawing the magnet the wire encloses none of it, all of the flux must have cut across the wire at some time during the withdrawal of the magnet.

If the coil around the magnet consisted of more than one turn of wire, the throw of the galvanometer would have been correspondingly greater. In other words, the galvanometer measures the "flux turns," i.e., the actual flux multiplied by the number of turns of wire across which it is cut.

123. Magnetomotive force is the cause of magnetic flux. It may be due to electric currents or to permanent magnets. Like its analog, E.M.F., which is measured by the work per

[1] Electrical Review, vol. 47, p. 441, 1900.

unit charge required to carry a quantity of electricity once around the electric circuit—magnetomotive force is measured, in precisely the same way, by the work per unit pole required to carry a magnetic pole once around the magnetic circuit. This unit of M.M.F. is often called a *gilbert.*[1]

Definition.—A gilbert is the magnetomotive force in a circuit when one erg per unit pole is required to carry a magnetic pole around the circuit.

It is not necessary to actually take the unit pole around the circuit, for if we know enough about the circuit it will be possible to compute the number of ergs of work that would be done in carrying the pole along the given path, and thus to determine the value of the M.M.F. This computation is especially simple when the magnetic forces are due solely to electric currents, for from the definition of unit current the work to carry a unit pole once around any closed path is 4π times the current enclosed by the path. Therefore the magnetomotive force in the complete circuit is,

$$\text{M.M.F.} = 0.4\pi NI \text{ gilberts,}$$

where I is the current in amperes and N is the number of times this current is linked with the magnetic circuit. The product NI is called "ampere turns" and differs from M.M.F. only by the constant factor of $0.4\pi = 1.26$. It is evident that a small current in a coil of many turns will produce the same magnetomotive force as a large current in a coil of few turns.

124. Reluctance, as magnetic resistance is called, corresponds to electrical resistance, and it depends in the same way upon the dimensions of the circuit and the material of which it is composed. Iron in its various forms is one of the best conductors of magnetic flux. Most other substances are rather poor conductors, but there are no magnetic insulators.

Although the laws of the electric and the magnetic circuits are similar in many respects, it must not be supposed that the analogies hold throughout. It requires no energy to maintain the magnetic flux when once established. There is therefore

[1] Sometimes a slightly larger unit of *one ampere turn* is used.

no analogy to Joule's law for the energy continually being dissipated in heat during the flow of current. There is a difference, too, between electric resistance and magnetic reluctance inasmuch as the former is constant for all ranges of current, while the value of the reluctance of a magnetic circuit depends upon the value of the magnetic flux.

If two paths are open to the magnetic flux it will divide just as an electric current would do between two wires in parallel. In a circuit made up of masses of iron of different dimensions and qualities, the total magnetic reluctance can be computed by summing up the separate reluctances as in the case of electrical resistances. The unit of reluctance is called an *oersted.*

Definition.—An oersted is the reluctance in a magnetic circuit when one gilbert is required to produce a flux of one maxwell in the circuit.

Problems

1. What M.M.F. is produced by a solenoid of 95 turns of wire through which flows a current of 5 amperes? 596.90 gilberts.

2. An iron ring is uniformly wound with 250 turns of wire. A current of 4 amperes produces a magnetic flux of 800,000 maxwells. Find the reluctance of the ring. 0.00157 oersted.

125. Flux from a Permanent Magnet.—The following exercises will illustrate the manner in which magnetic flux is measured. They may also serve to make clear some points that perhaps have not been fully understood from merely reading the theory of the subject. In regard to the second one, it is important that the facts be impartially observed and that a full and sufficient reason therefor be thoroughly understood.

A. Place a coil of one turn about the middle of a bar magnet. Note the deflection of the ballistic galvanometer when the magnet is quickly withdrawn. Repeat, using two, three, five, and ten turns.

B. Does it make any difference whether the coil is drawn off the north or the south end of the magnet?

C. Place the coil just off one end of the magnet and suddenly withdraw the latter. Note the deflection. Repeat, moving

the coil instead of the magnet. Repeat again, moving the coil at right angles to the magnet so as to "cut" across the magnetic flux. Is it true to say that the deflection depends only upon the total change in the flux threading the coil, and not at all upon the manner in which that change is produced?

126. Study of a Magnetic Circuit—Bar and Yoke.—The bar and yoke apparatus consists of a heavy rectangle of iron, through the center of which is the bar to be studied. This bar is divided in the middle and the two ends surfaced to fit

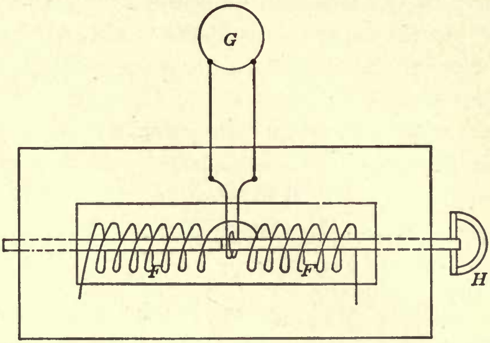


FIG. 71.—Bar and yoke.

well together. One-half of the bar is fixed rigidly in place while the other half can be withdrawn by the handle, *H*. Around the bar near the middle is a small bobbin on which is wound the coil which is joined to the ballistic galvanometer. When the movable part of the bar is withdrawn, this bobbin flies out to one side, thus carrying the coil into a region of no, or very small, magnetic flux. Thus the deflection of the galvanometer shows the total flux through the bar before it was parted.

The magnetizing current is passed through the coils *FF*, which extend the entire length of the bar, save for the short portion occupied by the bobbin at the middle. The bar is thus subjected to a uniform magnetizing force throughout its entire length. No magnetic poles are produced since the flux

through the bar completes its circuit through the yoke, the reluctance of which is small in comparison with that of the bar. Thus the length of the magnetic circuit for all practical purposes is equivalent to the length of the bar alone.

Starting with a small current through the magnetizing coils, the corresponding deflection of the galvanometer is determined. Then the current is increased by small steps and the corresponding deflections are noted, till the iron is fully magnetized. If the bar has been previously magnetized it will be necessary to demagnetize it before using it. This can be done by starting with the largest current previously used and decreasing it by steps while at the same time it is slowly reversed in direction eight or ten times at each step.

127. Computation for Magnetomotive Force.—The M.M.F. is readily computed from the value of the current by the formula,

$$\text{M.M.F.} = 0.4\pi NI.$$

where I is the value of the current expressed in amperes.

128. Measurement of Magnetic Flux.—The magnetic flux is measured by the deflection of the galvanometer, the latter being directly proportional to the amount of flux cut by the wires on the bobbin when it flies out from its position around the bar. If ϕ denotes the total flux through the bar, and n the number of turns of wire on the bobbin, the effect on the galvanometer is the same as cutting a flux of ϕn with a single wire. Therefore,

$$\phi n = cd,$$

where d is the deflection in scale divisions, and c is the *magnetic ballistic constant* of the galvanometer expressed in flux turns per scale division. The actual flux through the bar is, then,

$$\phi = \frac{c}{n} d.$$

129. To Find the Constant *c*.—The constant c is readily determined if a standard magnet is at hand. This is a steel

magnet the total flux through which is known. Call this Φ maxwells. A coil of n' turns is placed around the middle of the magnet and connected to the galvanometer. When the magnet is withdrawn the flux Φ is cut n' times, and as before,

$$\Phi n' = c d',$$

from which,

$$c = \frac{\Phi n'}{d'}.$$

130. Plotting the Results.—The results obtained from this study of a magnetic circuit can best be shown by means of curves. The first one may be plotted taking values of the M.M.F. as the abscissæ, and the corresponding values of magnetic flux for the ordinates.

From the values of the flux and the M.M.F. compute the reluctance of the bar for different values of the flux. Express the results by means of a second curve, this time taking the flux for abscissæ and using the corresponding values of the reluctance for ordinates.

131. Standard Curve—Unit Circuit.—Inasmuch as the flux through a magnetic circuit depends upon the length and cross section of the circuit, as well as upon the M.M.F., the curve just drawn shows the relation between the flux and the M.M.F. for this particular circuit only. If it is desired to find this relation for other circuits of the same material the results obtained above must be expressed in terms of a "unit circuit." That is, we can imagine a bit of the circuit 1 cm. long and 1 sq. cm. in cross section, say a centimeter cube, at some part of the circuit. Let us now draw a flux-M.M.F. curve for this unit circuit. This will be independent of the dimensions of the whole circuit, and will represent only the characteristics of the material being examined.

The ordinates for this curve, that is, the flux passing through the unit circuit, can easily be computed from the values of the total flux through the whole circuit. It is simply the total flux divided by the cross section of the circuit. This quotient,

which is the flux per square centimeter, is usually denoted by the letter B, or in symbols,

$$B = \frac{\phi}{A},$$

where A denotes the cross section of the circuit in which the total flux is ϕ.

Similarly, the abscissæ are computed from the values of the total M.M.F. Since the unit circuit is 1 cm. long the M.M.F. in this length will be the total M.M.F. divided by the length of the entire circuit—provided the latter is uniform. This quantity, which is the M.M.F. per linear centimeter, is usually denoted by the letter H, or in symbols,

$$H = \frac{\text{M.M.F.}}{L}.$$

where L is the length of the circuit.

In the case of the bar and yoke the reluctance of the yoke is small compared with that of the bar, and therefore the equivalent length of the whole circuit is practically that of the bar alone.

Such a B-H curve should now be drawn in addition to the two already plotted for this circuit.

CHAPTER XI

COMPLETE DEFINITION OF THE MAXWELL

132. Complete Definition of the Value of Unit Flux.—The preceding experiment furnishes a good and complete study of a magnetic circuit, and the curves just obtained show the characteristics of the material composing the circuit. But referring back to the discussion of magnetic flux it will be found that the value of a maxwell of flux was there stated, but it has not yet been defined. If anything more than an empirical rule of thumb is desired, it will be necessary to find a definition of unit flux.

Why can not the statement in Article 122 regarding the value of one maxwell be taken as a definition, and thus fix the amount of flux that should be taken as unity? Because in the definitions for unit current and for unit magnetic pole there has already been given, implicitly, the value of unit flux. And the only consistent thing that can now be done is to determine, explicitly, just how much flux has thus been fixed at unity. This will require a careful review of all the definitions and other relations between magnetic poles and electric currents. The discussion is straightforward, except as we may pause occasionally by the way to note some interesting or important facts.

We shall therefore now take up in logical order a series of definitions and propositions leading to the discovery of the value of unit flux, and the establishment of the relations from which will follow the statement heretofore given.

133. Relations Between Current and the Magnetic Field.—The definition for unit current gives the relation that the amount of work required to carry a unit magnetic pole once

around an electric current is $4\pi I$ ergs. For a pole of strength m the work would be $4\pi Im$ ergs. Since this is true for all cases, it will be true for the particular case of a long straight current with the pole carried around it in a circle of radius a perpendicular to the conductor, the center of the circle being at the center of the conductor. At each point in the path the pole is always at the same distance from the current, and whatever force it may experience at one point will be, by symmetry, the same at every other point in its path. Let F denote the value of this force. The length of the path is $2\pi a$. Therefore the work required to carry the pole once around the current against this force is,

$$W = 2\pi aF.$$

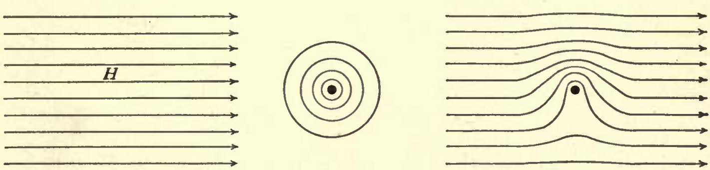
But from the definition of unit current this is also,

$$W = 4\pi Im.$$

Hence the force experienced by the pole at each point of the path is,

$$F = \frac{2Im}{a} \text{ dynes.}$$

134. Force between Current and Magnetic Field.—When a conductor carrying a current is also in a magnetic field, there



(a) (b) (c)

FIG. 72.—The magnetic field surrounding an electric current.

is a force acting upon the conductor tending to move it across the field. This force is due to the mutual action between the field produced by the current and the original field which exists independently of the current.

Suppose the original field is uniform and constant when no

current flows through the straight conductor under consideration: This condition can be represented as in Fig. 72*a*. The field around a current would be represented by a series of concentric circles with centers on the conductor as shown at *b*. If now these fields both exist at the same time, the sum of the two is as shown in *c*, and the conductor is urged downward. Evidently this effect is greatest when the current is at right angles to the direction of the magnetic field. The exact value of the force acting on the conductor can be determined from the following considerations.

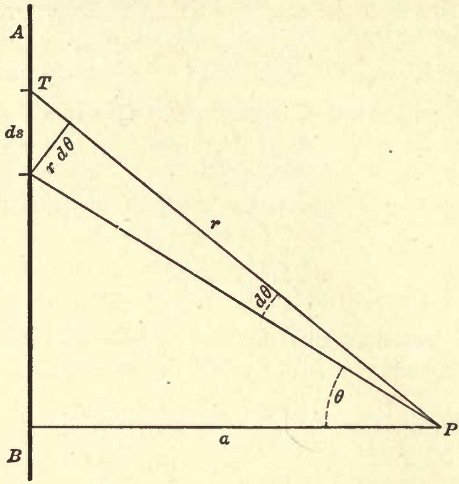


FIG. 73.

It is shown above that the force on a pole, m, at P, due to a current in the long straight conductor AB, is

$$F = \frac{2Im}{a}. \tag{1}$$

Since action and reaction are equal, the conductor itself must experience an equal and opposite force tending to move it at right angles to the plane APB, and which, in terms of the polar theory of magnetism, would be said to be due to the pole

at P. But as there can be no "action at a distance," or as Lord Kelvin used to put it, "action by matter where it is not," it would be more consistent to express this force in terms of the magnetic field immediately surrounding the conductor. And although in reality all electric currents flow in closed circuits, yet for purposes of computation it is often convenient to deal with portions of a circuit. For this reason, and in this sense, the total force experienced by the conductor AB may be considered as the sum of all the forces on each part of the wire.

Thus let us consider an element of length ds at any point T. The intensity of the magnetic field at T due to a pole of strength m at P is

$$H = \frac{m}{\mu r^2} \tag{2}$$

and the component of this, perpendicular to the wire AB is

$$H' = \frac{m}{\mu r^2} \cos \theta \tag{3}$$

It is natural to suppose that the force acting upon an element of the conductor at T will depend upon the amount of current in the wire, the intensity of the magnetic field and the length of the portion considered.

That is, if f denotes the value of this force,

$$f \ \alpha \ (I, H', ds)$$

or,

$$f = kIH'ds \tag{4}$$

where k is the proportionality factor. The force on the whole wire will then be the summation of the forces acting on each separate portion of it. In Eq. (1) we see what this summation must be. We also see that m appears in this summation in the first power; therefore each of the other factors in H' must also enter in the first power.

The force acting upon the entire length of this conductor is, then,

$$F = k\int_{-\infty}^{+\infty} \frac{Im \cos \theta \, ds}{\mu r^2} \tag{5}$$

The three variables can be expressed in terms of one by observing from similar triangles that

$$\frac{ds}{r} = \frac{rd\theta}{a} \tag{6}$$

which gives,

$$\mathrm{F} = \frac{kIm}{\mu a}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\,\theta\, d\theta = \frac{2kIm}{\mu a} \tag{7}$$

This agrees with (1) if $k = \mu$. Therefore the relation in (4) should have been

$$f = \mu H'Ids = \mathrm{B}'Ids \tag{8}$$

as the true expression for the value of the force acting upon an element of the conductor.

135. Magnetic Induction.—We are here introduced to a new quantity which now for the first time enters our equations in explicit form. This is the quantity μH, which often appears in this way, the two factors entering thus together. This product has distinctive properties which are quite different from those of H alone, the same as the product RI is not like I. And as in the latter case, the letter V is often written for RI, and the quantity thus designated is given a distinctive name, "fall of potential," so in the former case the product μH is usually represented by the single letter B, and the quantity thus designated is called the "magnetic induction."

As H denotes the intensity of the magnetic field and has different values from point to point, so B has a definite value at each point. The value of H is measured by the force which acts upon a magnetic pole; the value of B is measured, Eq. (8), by the force which acts upon a conductor carrying an electric current. Either quantity may be represented by lines drawn through each point in space in the direction of the force at that point. If more lines are drawn where the force is strong, and fewer where the force is less, the density of these lines, that is, the number per square centimeter, may also be made to represent the numerical value of H, and of B, respectively. This idea of drawing lines to represent these quantities was

introduced by Faraday, who used them thus to visualize the state of the medium in the neighborhood of a magnet or a current. For the case of $\mu = 1$, it is evident that the map for H would be similar to the map for B; for other values of μ the maps for the two would be different.

136. Force Exerted on a Conductor Carrying a Current.—Equation 8, above, is a very important relation. In the first place it shows that the force acting on the conductor does not depend solely upon H, the intensity of the magnetic field, but it is proportional to μH, which is quite a different thing. Furthermore, if a finite length L of the conductor crosses a flux of uniform density B, the force acting upon this length is

$$F = B'IL, \tag{9}$$

where B' (from Eq. 3) is the component of the magnetic induction at right angles to the direction of L.

If in addition to being uniform, the magnetic induction is at right angles to the current, then a wire L centimeters in length will be urged across the field with a force,

$$F = BIL \text{ dynes.} \tag{10}$$

This force acts upon the conductor as a whole, and not upon the current which it carries. Several investigations have been made to determine whether the current is urged to one side of the conductor, but no effect of this kind has ever been detected.

Definition.—Unit magnetic induction is that magnetic induction in which a conductor carrying unit current (= 10 *amperes) experiences a force of one dyne for each centimeter of length.*

This definition does not lend itself, directly, to the measurement of magnetic induction because it is not easy to measure small forces with accuracy. But it does lead at once to a simple method for measuring the total magnetic flux, from which the magnetic induction is readily obtained, if desired, by computation, as is shown below.

137. Work when a Current Moves through a Magnetic Field.—Let a conductor such as that supposed in (10) above,

and carrying a current I, move a distance s in the direction in which it is urged. The work which can be done by F is

$$W = Fs = BLIs, \tag{1}$$

and this energy is supplied by the source that is maintaining the current.

As explained above, B denotes the value of the magnetic induction at a given point. When multiplied by a definite area, normal to the direction of B, the product is the total induction, or the magnetic flux, through this area. Since Ls is the area swept over by the conductor, the product BLs is the magnetic flux, ϕ, cut by the conductor in its motion. Therefore,

$$W = \phi I \tag{2}$$

from which it appears that one erg of work is required to move a conductor carrying unit current across a magnetic flux of one maxwell.

138. Induced Electromotive Force.—The electrical expression for work is

$$\begin{aligned} Work &= EIt = (e' + e)\,\mathrm{I}t \\ &= RI^2t + eIt, \end{aligned}$$

where E is the E.M.F. required to maintain the current I, and t is the time that the current is flowing, measured in seconds. A part the energy is used in merely keeping the current flowing through the resistance in the circuit in accordance with Ohm's law; and this part appears as heat. If any other work is done it is included in the part

$$W = eIt,$$

where e is the extra E.M.F. required to do the work. Since e is not constant it is better to write,

$$W = \int eIdt = \phi I. \tag{3}$$

Equating (1) and (3),

$$\int edt = BLs = \phi. \tag{4}$$

Differentiating this gives for the value of e,

$$e = \frac{d\phi}{dt} \tag{5}$$

Therefore the movement of the conductor across the magnetic flux has induced in the conductor itself an equal and opposite E.M.F., viz.,

$$e = -\frac{d\phi}{dt} \tag{6}$$

and it is to counter balance this induced E.M.F. that it is necessary to supply the extra E.M.F. to keep the current from decreasing.

Since Eq. (6) is independent of the value of the current in the wire, e will have the same value when the current is zero. Therefore when a wire carrying no current is moved across a magnetic flux there is induced in it an E.M.F. the same as above, and if the circuit is closed this will cause a current to flow. The direction of this current will be opposite to that of the steady current considered in (3).

139. Definition of a Maxwell.—As stated in Article 122, the unit of magnetic flux is called a maxwell. Equation (6) now shows that a magnetic flux can be measured by means of the E.M.F. induced in a conductor, and this relation furnishes the basis for defining the value of a maxwell.

Definition.—A maxwell is the magnetic flux cut each second by a conductor in which there is induced an E.M.F. of one C.G.S. unit.

The name "maxwell" for unit magnetic flux was adopted by the International Electrical Congress at Paris in 1900.[1] Where 10^8 maxwells are cut each second the induced E.M.F. is one volt.

Corollary.—Unit Flux Density. A uniform flux of one maxwell through each square centimeter normal to the direction of the flux would be a flux density, or magnetic induction, of unity.

Equation 10, Article 136, also gives the value of this unit

[1] See Elec. Rev., Vol. 47, p. 441. 1900.

expressed in terms of the force exerted upon a conductor carrying a current.

140. Measurement of Magnetic Flux.—Having seen above that an E.M.F. is induced in a wire when the latter cuts across a magnetic flux, it is at once evident that this offers a ready means for the measurement of magnetic flux. As this E.M.F. exists only while the flux is being cut, it gives rise to a transient current, the total quantity in which can be measured by a ballistic galvanometer, as shown below.

The equation of a ballistic galvanometer is,

$$Q = kd,$$

where k is a constant and d is the first throw of the needle corrected for damping. If the galvanometer is connected to the wire when it is moved across a magnetic flux, the total quantity of electricity that passes through the circuit is,

$$kd = Q = \int idt = \int \frac{e}{R}dt = -\frac{1}{R}\int_{\phi'}^{\phi''} d\phi = \frac{\phi' - \phi''}{R} \qquad (7)$$

where ϕ'' denotes the amount of flux enclosed by the galvanometer circuit at the beginning, and ϕ' the amount at the end of the motion. Hence the amount of flux cut across by the wire is,

$$\phi' - \phi'' = Rkd \qquad (8)$$

where R is the total resistance of the galvanometer circuit.

Hence the quantity measured by the ballistic galvanometer in Article 128 is what has been denoted above by ϕ, and called magnetic flux. $(= \mu HA.)$

141. Relation Between Field Intensity and M.M.F.—Since a magnetic pole in a magnetic field is under the action of a force, whenever such a pole is moved work must be done, either positive or negative according to the direction of the motion. The longer the path and the stronger the field the greater will be the expenditure of work.

By definition, the magnetomotive force in any circuit is measured by the work per unit pole to carry a north pole once

along the circuit. If the magnetic intensity has the value H at each point along the path, it is evident that the work per unit pole and therefore the magnetomotive force, is,

$$\int HdL = M.M.F.$$

where L is the length of the path. In case H has the same value at each point this "line integral" becomes simply HL.

Therefore if enough is known about the circuit to determine the values of H, the M.M.F. can be computed. Or, if the M.M.F. is known it is possible to compute the value of H in some simple circuits. A few such examples are given below, including those of most importance in practical measurements.

142. Long Straight Current.—In the case of a long straight conductor carrying a current it is not difficult to compute the intensity of the magnetic field at a distance a from the conductor. Since the work per unit pole required to carry a magnetic pole around the current by any path whatsoever is $4\pi I$, let the path be a circle of radius a. By symmetry, the magnetic force is constant along this path, and therefore

$$4\pi I = H \times 2\pi a \tag{1}$$

where H denotes the intensity of the magnetic field.

From this,

$$H = \frac{2I}{a} \tag{2}$$

143. Ring Solenoid.—Another case that can be easily solved is that of a uniform spiral, or helix, of many turns of wire and bent into a circle so as to bring its ends together. This is the case of a ring uniformly wound with N turns of wire. The magnetic field is all within the spiral forming the ring and in the direction of its length. It will therefore require work to carry a magnetic pole around the ring, for in making the complete circuit the pole has passed around a total current of NI. The total work per unit pole is, then,

$$\frac{W}{m} = 4\pi NI.$$

If the path is a symmetrical circle of radius r, the magnetic forces will have the same value, H, at each point of the path. Hence if L is the length of the path,

$$W = HmL,$$

and

$$H = 4\pi \frac{N}{L} I = 4\pi n I,$$

where n denotes the number of turns per centimeter.

Since $L = 2\pi r$, where r is the radius of the path,

$$H = \frac{4\pi NI}{2\pi r} = \frac{NI'}{5r},$$

where I' is the value of the current in amperes. This shows that the intensity of the magnetic field is not constant across the section of the ring, but varies inversely as r, being greater on the inner side of the ring.

144. Long Straight Solenoid.—A long straight solenoid, wound uniformly with n turns per centimeter, may be considered as a portion of a ring solenoid of very great radius. At points within the solenoid, and not near the ends, the magnetic forces are practically the same as though the entire ring were present. From the preceding section the value of the magnetic intensity is, then,

$$H = 4\pi nI.$$

Or, if the current is measured in amperes

$$H = 1.2566nI'.$$

145. Magnetic Field due to Any Electric Current.—In the examples considered above the magnetic field due to a current could be very easily calculated. In most cases the calculation is not so simple. But if we may make the assumption that each linear element of current, independently of the other elements of the circuit, contributes its own share toward making up the total field, it is possible to find an expression for the field due to a single element which, *when integrated over the entire*

length of the electric circuit, will give the correct value for the intensity of the magnetic field at any given point. This will enable us to calculate the magnetic field due to a current in any circuit whatever. The results so obtained have been found to agree with the facts in every case where the magnetic field can be measured by other means. This is not to be taken as proving what the effect of a single element would be, if such an element could exist by itself, but as showing that the formula will give correct results when applied to any actual current.

146. Magnetic Effect of a Current Element.—Let ds, Fig. 74, represents a short element of an electric circuit. It is required to find the intensity of the magnetic field at any point, P, due to a current I flowing through ds.

By the definition of Article 3 the intensity of the field at P is measured by the force per unit pole which would be exerted upon a magnetic pole if one were placed at P. That is,

$$dH = \frac{F}{m}, \tag{1}$$

where dH denotes the part of the total intensity that is due to the element ds. Therefore let us suppose there is a pole m at P, and then let us determine the force which it would experience. Finding F, the value of H is readily computed.

In Eq. 8, Article 134, above, the force which a linear current element would experience in a magnetic field, H', at right angles to itself, was found to be

$$f = \mu H'Ids \tag{3}$$

In order to express this force in terms of the pole m, the value of H' as found on page 163 may be substituted in (3). This gives

$$f = \frac{mIds \cos \theta}{r^2}. \tag{4}$$

Since action and reaction are equal, if the element ds experiences this force because of the presence of m, then m will

experience an equal force of reaction from ds. Therefore the force acting upon m is,

$$F = \frac{m\,I\cos\theta\,ds}{r^2} \tag{5}$$

and the part of the magnetic field at P contributed by the element ds is,

$$dH = \frac{F}{m} = \frac{I\,ds\cos\theta}{r^2} \tag{6}$$

The expression $ds \cos \theta$ is readily seen to be the component of ds normal to r.

147. Extension to General Case.—To find the value of H at any given point P, due to a current in a given circuit, let P, Fig. 74, be taken as the origin of polar coördinates, r and θ. Then the integral of (6) taken along *the entire length of the electric circuit* will give the correct value for the field intensity at P. The computation will be simpler, in general, if the entire circuit and P are in one plane, for then all of the components, such as dH, will be in the same straight line through P and normal to the plane, and the resultant will be merely their sum.

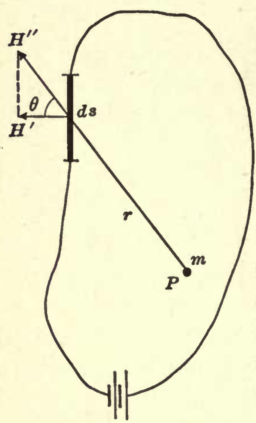


FIG. 74.

If the various elements, dH, do not lie in the same line, the resultant field in any direction is the sum of the components of the various dH's resolved in the given direction.

148. Magnetic Field at Center of a Circle.—In the case of a circular loop of current with the point P taken at the center of the circle, the above integration can be performed quite simply. Each element, ds, of the conductor is now at the same distance, r, from P, and since ds is always at right angles to r, $\cos \theta$ is unity for each element. Since P lies in the plane of the circle,

the resultant magnetic force is simply the arithmetical sum of the effects from each element of the current. Therefore,

$$H = \int_{o}^{2\pi r} \frac{I}{r^2} ds = \frac{2\pi I}{r}$$

for the entire circle.

Corollary.—The common definition of unit current follows at once from this as that current which will produce at the center of a circle of unit radius, a magnetic field of unit intensity for each centimeter length of the current.

Problem.—Given a current of I amperes flowing in a circle of radius a cms. Find the value of H at a point on the axis, $2a$ cms. from the plane of the circle.

CHAPTER XII

MAGNETIC TESTS OF IRON AND STEEL

149. Introduction.—In the preceding chapter there was given one method for determining the magnetic qualities of a piece of iron. The bar and yoke method is very useful for purposes of illustration and instruction, since it is readily understood that the wire on the little bobbin cuts across the magnetic flux when it is released by the withdrawal of the bar. But the air gaps at the end of the bar and at the side where it slides in the yoke introduce considerable reluctance into the magnetic circuit, and therefore the magnetic flux is smaller than it would be for a similar circuit containing no air gaps. Such closed circuits are used in the methods described below, and while it is not possible for the wire to cut across the flux, yet when the flux is changed from one direction to the reverse it is evident that the change in the flux through the coil has been twice the original amount. In fact, if the bobbin in the bar and yoke had remained around the bar while the magnetizing current is reversed, the deflection of the galvanometer would have been twice the amount that it was when the bar was withdrawn.

The following methods are of varying intrinsic value, but they are described in the given order as each one leads up to the one that follows. There is sufficient repetition to emphasize the important points and to make each method intelligible when that section only is read. The methods given in Articles 150, 155, and 158, are most frequently used.

150. Double Bar and Yoke.—This apparatus consists of two massive yokes of Swedish iron fitted to carry two round bars of the iron or steel to be tested. When clamped together the

bars and yokes form a rectangular circuit, the bars forming the longer sides. Over the entire length of each bar is a brass spool on which is wound 300 turns of wire to carry the current used to magnetize the iron. The joints where the bars pass through the yokes are carefully fitted to avoid any unnecessary reluctance at these places. Thus the magnetic circuit consists of the two bars and very little else since the reluctance of the yokes is small in comparison.

This method differs from Hopkinson's bar and yoke in that the bar can not be pulled open to allow the test coil to fly out and cut the flux it is desired to measure. Therefore the test

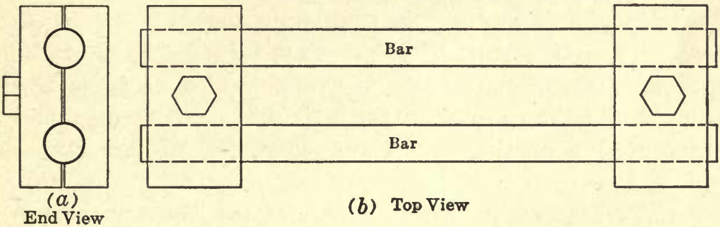


FIG. 75.—Double bar and yoke, without the magnetizing coils.

coil is wound around the middle of the bars, and the flux through the circuit is reversed by reversing the magnetizing current. This gives twice the change of flux through the coil and therefore twice the deflection that would be obtained by simply cutting the flux once.

In making a test by this method it is well to start with a magnetizing current of about one ampere. This should be reversed ten or more times, allowing the current to rise to its full value after each reversal. On the last reversal the galvanometer circuit can be closed and the deflection observed. This should be repeated twice, or until consistent readings are obtained, closing the galvanometer circuit only when the reversal is the same way as before in order to keep the galvanometer deflection always in the same direction. The current can now be reduced about 10 per cent. and after eight or ten slow

reversals, as before, the corresponding deflection can be observed. In this way the current is reduced to zero by ten or fifteen steps, which should be shorter when going over the steep portion of the curve. If when adjusting the current to a new value it falls too low it can be raised to the desired value and no harm is done. On the other hand, should the current, even for an instant, reach a larger value than the one last used it will be necessary to reduce it again by small steps and many reversals in the same manner as was done the first time.

The results of this test are best shown by a magnetization curve. This could be plotted between M.M.F. for abscissæ and flux for ordinates as was done in a previous experiment. Or better still, and as is more commonly done, by plotting the B–H curve. This curve has the same form as the other, but since the dimensions of the bar are divided out, the B–H curve shows the quality of the iron while the former curve represented a particular curcuit composed of this kind of iron.

151. To Determine the Constant of the Galvanometer.—The magnetic ballistic constant of the galvanometer and the test coil can be determined by means of a standard magnet as explained in Article 129, or by using a "standard coil." This is a mutual inductance consisting of two coils wound upon the same spool. One of these coils is placed in the galvanometer circuit and the other may be connected to a battery when it is to be used.

Let I' denote the current through the primary when the circuit is closed. Then the flux-turns through the secondary is MI' (see Article 171), where M is the mutual inductance (C. G. S.) of the given pair of coils, and is numerically equal to the flux threading the secondary when unit (C. G. S.) current flows in the primary. Each time the primary current, I', is made or broken, the secondary is cut by this flux, causing a fling d' of the galvanometer.

If c denotes the magnetic ballistic constant, that is, the flux cut per scale division of deflection, we can write,

$$cd' = MI' \qquad \text{or,} \qquad c = \frac{MI'}{d'} \quad .$$

152. To Find the Value of B from the Deflection.—In studying a given sample of iron or steel we are usually not so much interested in the properties of the particular piece under investigation as in the specific properties of that grade of iron. The actual total magnetic flux, ϕ, depends as much upon the dimensions of the sample studied as upon the quality of the iron. If the cross section of the iron is A sq. cm. and the flux density is B, then $\phi = BA$ maxwells. It is this quantity B which depends upon the quality of the iron, and in which therefore we are most interested. Since ϕ is measured in maxwells, B is given in maxwells per square centimeter, and is called "the magnetic induction." It is also called "flux density."

From what has been said thus far it will be seen that the magnetic flux can not be measured directly. It is only the *change* in the *flux turns* that affects the galvanometer; and the value of the total flux must be inferred from such measurements. The most usual change of flux is that produced by reversing the magnetizing force. It is then assumed that the flux is also reversed, and therefore the change produced is twice the total flux, Then,

$$\text{Change in flux turns} = 2\phi n = 2BAn = cd,$$

or

$$B = \frac{cd}{2An} = Jd.$$

All of these factors can be found and the numerical value of J computed once for all. If it is possible to adjust the resistance of the galvanometer circuit it simplifies the computation to make $J = 100$. The values of B can then be obtained as fast as the deflections can be read and multiplied by J.

153. To Determine the Value of *H*.—The M.M.F. required to magnetize a bar of iron to a certain degree depends directly

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upon the length of the bar. Therefore if we are studying the properties of the material rather than those of the bar as a whole, the applied magnetizing force is expressed in terms of the M.M.F. per centimeter length. Thus,

$$H = \frac{\text{Total M.M.F.}}{\text{Total length}} = \frac{1.26\,\text{NI}'}{L} \text{ gilberts per centimeter.}$$

154. Permeability.—As in the case of electric conductors the conductivity of a substance is,

$$C = \frac{I/A}{E/L} = \frac{\text{Current density}}{\text{Volts per cm.}}$$

so in magnetism the permeability of a substance is given by the similar relation,

$$\text{Permeability} = \mu = \frac{\phi/A}{M.M.F./L} = \frac{B}{H} = \frac{\text{Flux density}}{\text{Gilberts per cm.}}$$

Permeability curves should also be drawn for each kind of iron, using the "magnetic induction," B, or flux density, for abscissæ and the corresponding values of the permeability for ordinates.

155. The Ring Method.—In some respects this method is preferable to the bar and yoke. The sample of iron under investigation is in the form of a ring, and has no ends and no joints. This ring is wound uniformly along the entire length with one or more layers of wire for the primary or magnetizing coil. Over this is wound the secondary, usually consisting of a few turns, which may be either well distributed or bunched at one place. Since the primary turns are closer together on the inside of the ring than on the outside, the magnetizing force will not be uniform within the iron, but will be stronger near the inner side. For this reason it is best to use a broad and flat ring, shaped like a wagon tire. In selecting the iron, care must be used to obtain a homogeneous bar. Rowland suggests that it is better to have it welded than forged solid; it should then be well annealed and *afterward* have the outside taken off *all round* to about one-eighth of one inch deep in a

lathe. This is necessary, because the iron is "burnt" to a considerable depth by heating even for a moment to a red heat, and the permeability of this portion is therefore unlike the rest of the ring.

If the ring is a new one which has never been magnetized, the smallest currents should be first used. Great care must be observed that at no time does the current exceed, even for an instant, the value being used at the time. If the iron has been previously magnetized it must be thoroughly demagnetized before it is used.

From the magnetizing current and the length of the ring, the value of the average magnetic force within the iron is easily computed from the expression

$$H = \frac{4\pi NI}{10\,L} = \frac{NI}{5a}$$

where a is the mean radius of the ring.

From the galvanometer deflection and the cross section of the ring the magnetic induction is

$$B = \frac{c\,d}{2\,n\,A}$$

as shown in Article 152.

The results of this experiment are best shown by means of a curve, plotting for abscissæ the values of H, and for ordinates the corresponding values of B.

156. The Step by Step Method.—This method is much like the preceding, the only difference being that the magnetizing current is not reversed to give the galvanometer reading. Instead of this the current is constantly maintained in one direction. When the current is first turned on, the iron is magnetized to the corresponding amount and if the galvanometer circuit is closed there will be a proportional deflection. When the galvanometer has returned to its zero position the current is suddenly increased to its next value. The corresponding deflection of the galvanometer measures not the actual magnetism of the ring, but the *increase* over the former

amount. In the same way the current is increased by steps until its maximum value is reached, while each of the corresponding deflections are carefully noted. The actual magnetism of the ring at any stage is measured by the *sum* of all the deflections up to that point. If this sum is denoted by Σd, the magnetic induction is

$$B = \frac{c\Sigma d}{nA}$$

Since the ammeter in the primary circuit gives the total current at any point, the expression for the magnetic force will be the same as before.

$$H = \frac{4\pi NI}{10\,L}$$

The *H-B* curve plotted from these values should be the same, very closely, as that obtained by the ring method using reversals of the current.

157. Hysteresis—Step by Step Method.—In the step by step method just described great care was observed never to have a larger current in the primary coil than that being used at the time, and the magnetization curve was obtained by always using increasing values of the current.

Suppose that after reaching the maximum the current should be *decreased* by steps, and the corresponding deflections noted. Would the magnetization curve be retraced, or would a new curve be obtained? As the current is slowly removed it would be found that the magnetization of the iron does not decrease to its former values, and when the current is reduced to zero there still remains a large amount of "residual" magnetism. This return curve can be traced perfectly well by the step by step method, and it is shown by the curve ,*AD* Fig. 76.

It will even require the application of a reversed magnetizing force, equal to *CG*, to reduce the magnetization to zero. This value of *H* is called the coercive force of the iron. It is large for hard iron, and steel, but small for soft iron and silicon-iron alloys. If the reversed field is increased to a value

CF, equal to *CE*, the iron will be magnetized as strongly as before, but in the opposite direction, and it will hold this magnetization just as persistently as the other. If *H* is reduced to zero and again increased to *CE*, the magnetization follows as shown by the curve *PJA*. This lagging of the values of *B* behind the corresponding changes in *H* is called hysteresis, from a Greek word meaning "to lag behind." The complete curve as thus drawn between *B* and *H* is called a hysteresis curve.

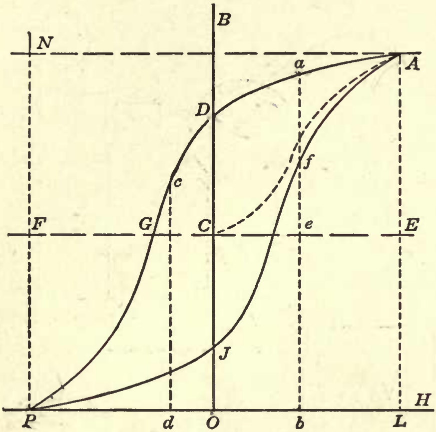

FIG. 76.—Hysteresis curve for tool steel.

For the determination of a hysteresis curve by this method the setup and manipulation would be the same as in the preceding experiment. And the results, plotted on a *B-H* diagram will give a curve similar to the one shown in Fig. 76.

158. Hysteresis by Direct Deflection.—Referring to Fig. 76, suppose that the iron has been carried around the cycle *ADPJA* several times and left in the condition represented by the point *a*. If then it is carried from *a* to *P* by a single step, the deflection of the galvanometer will measure the corresponding change in the magnetic induction, represented by the ordinate *ab*. By carrying the iron around the cycle to the point *a* again,

this reading can be repeated as many times as desired, or, by stopping at some other point near a, the corresponding ordinate can be determined. Thus the curve ADP can be traced.

Let the setup be made as shown in Fig. 77, where Z is the ring of iron to be studied. This iron is magnetized by a current from a few cells of the storage battery. The resistance R is set at the value which gives the maximum current that is desired. S is an ordinary double throw, two pole, switch which is made into a reversing switch by the addition of two diagonal con-

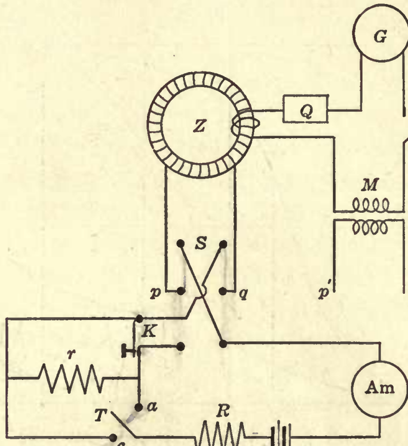

FIG. 77.—To determine the hysteresis curve for the ring, Z.

nections. As shown in the figure, one of these diagonal connections of S is formed by the adjustable rheostat r, which can be short circuited by closing K. The rheostat R can be connected to either end of the rheostat r by closing the switch T on either point a or point c. This introduces r into the battery circuit when S is up, or down, respectively.

With T closed on a.—When the switch S is thrown down, thus connecting the primary of Z directly to the battery circuit, the magnetic state of the iron is represented by the point P, Fig. 76. When it is thrown up, the current through Z is reversed,

but the current will be smaller than before because the extra resistance r is now in the circuit, and the iron will be brought to some point as f on the curve JA. By closing K the iron is brought to A, and when K is opened the iron comes to some point as a, depending upon the setting of r. The resistance in r must have been previously adjusted to the proper value in order that the iron may be brought to the desired point when K is opened. (In case r is adjusted after K is opened, the iron must be taken around the entire cycle several times before it is certain that the point a is reached.)

The galvanometer is joined in series with the secondary coil of the standard mutual inductance, M, a resistance box Q, and the few turns of wire forming the secondary winding on the ring. It is often convenient to use a shunt on the galvanometer, adjusted to give critical damping. If S is now thrown down, changing the value of H from $+Ce$ to $-CF$, the galvanometer deflection will measure the corresponding change in B, indicated by ab, Fig. 76. In order to get back to the top of the curve for another determination, S is thrown up bringing the iron to f on the lower curve. Then K is closed, thus carrying the iron on to A. Opening K brings the iron to the point on AD corresponding to the setting of r.

The cycle of operations is thus, (1) throw S down, (2) throw S up, (3) close K, (4) open K. The galvanometer is read during the first operation.

With T closed on c.—To locate points on the curve between D and P requires a slight change in the relative position of r. This is effected by changing the connection at T from a to c. Then when S is thrown up the iron is always at A, Fig. 76, and when thrown down the current will be reversed but less than its maximum value because r is now in the circuit. By setting r at the proper value the iron can be brought to any desired point between D and P. When K is closed the iron is carried on to P and the corresponding deflection of the galvanometer gives the ordinate cd.

The cycle of operations is now, (1) throw S up, (2) throw S

down, (3) close K, (4) open K. The galvanometer is read at the time of the third operation.

In all of these measurements the iron is carried to P, and thus the final magnetizing force is the maximum that has been used. This is better than ending with a weak value of H, because under weak fields the iron will not come to its final magnetization as promptly as under stronger fields.

The remaining portion of the curve, PJA, can be determined in precisely the same way. With K and S set so that the iron is at P, let the connections from S to the ring Z be interchanged. This will carry the iron to A. By turning Fig. 76 bottom side up it will be seen that the curve PJA corresponds exactly with the former curve ADP; and it can be traced by repeating the observations previously made, but the ordinates must now be plotted from the line AN. Or, since the curves are the same, the former set of ordinates can be used again for the second portion of the curve.

The actual plotting of the curve AJP downward from the line AN is often awkward, as usually AN does not coincide with one of the main divisions of the cross-section paper. An easier way is to lay a second piece of paper under the curve ADP and fasten both together by two pins through A and P Several points along the path of the curve are pricked through both papers with a needle point. The lower paper is then placed over the curve, with the points A and P interchanged, and fastened on the two pins. The intermediate points are then pricked through on to the other sheet, thus outlining the curve PJA.

159. Determination of the Values of B.—In this experiment the change in the magnetic induction is not a reversal, but a change from ea, Fig. 76, in one direction to eb in the other. This gives a total change of ab, which may be denoted by the symbol ΔB. The corresponding change in flux turns in the secondary circuit is ΔBAn, and this is measured by the galvanometer deflection, the relation being,

$$\Delta BAn = cd \tag{1}$$

The change in the magnetic induction is then

$$\Delta B = \frac{c}{An} d = Jd, \tag{2}$$

and this is plotted as the ordinate *ab*, Fig. 76, the values being laid off from the axis *OH*. After the curve is drawn the true axis, *FE*, is drawn through the middle of the figure.

160. The magnetic ballistic constant, c, may be determined by means of a known mutual inductance as in the previous methods. From Article 151,

$$MI' = cd'.$$

where I' is the change in the current through the primary of the inductance M, measured in *C.G.S.* units. Since $c = JAn$, from (2), the deflection of the galvanometer for any desired value of J is,

$$d' = \frac{MI'}{JAn},$$

Usually it is convenient to choose $J = 100$, and then adjust the resistance in series with the galvanometer until the deflection is d' for a change of I' *C.G.S.* units of current in the primary of the calibration coil.[1]

The secondary of this mutual induction must remain a part of the galvanometer circuit, as shown at M, Fig. 77, since any change of the resistance of this circuit will change the value of the constant.

161. The values of the magnetizing force, H, are determined by the current, which may be read by an ammeter. Values of H may then be computed as shown in Articles 153 or 155.

[1] NOTE.—Inasmuch as no current is used for the ring Z while the constant of the galvanometer is being determined, the same battery, ammeter, and rheostats can be used to furnish the current I'. To make this change it is only necessary to exchange the connections pq for $p'q'$, Fig. 77.

Since the zero position of a sensitive D'Arsonval galvanometer depends upon the direction in which it was last deflected, all deflections should be in one direction only, and therefore the constant must be determined for deflections in this direction also. If greater accuracy is desired the scale should be calibrated by determining the value of J for deflections throughout the range that will be used.

These values are plotted along the axis FCE, with C as the origin.

162. Energy Loss through Hysteresis.—Since it requires a reversed current to bring the magnetization of a ring or bar to zero, there is always a considerable loss of energy when a piece of iron is carried through a cycle of magnetic changes. This energy is represented by the area of the hysteresis loop, which is narrow for wrought iron while for cast iron it is large and broad.

The amount of energy thus transformed into heat can be determined as follows: The work required to magnetize the iron is partly lost as heat in the magnetizing solenoid. This is the regular Ri^2 loss. Another part is used in maintaining the current against the induced E.M.F. due to the newly formed magnetic field. This induced E.M.F. is

$$e = -N\frac{d\phi}{dt} = -\frac{ANdB}{dt},$$

where A is the cross section of the iron and N is the number of turns in the magnetizing solenoid.

Since e is negative, *i.e.*, opposed to the current, the positive work required to maintain the current is

$$W = \int eidt = \int \frac{ANidB}{dt}dt = \int \frac{AHl}{4\pi}dB = \frac{\text{Volume}}{4\pi}\int HdB$$

since $Hl = 4\pi Ni$.

For one complete cycle $\int HdB$ is the area of the hysteresis curve, measured in the units of H and B. This area, then, gives the energy expended per cycle in each 12.57 cc. of iron.

163. Permanent Magnets.—Referring to the hysteresis curve for the bar and yoke, it is seen that the bar is still magnetized after the current is stopped. When the bar is withdrawn from the yoke, it is in the same condition as before. The bar itself is a permanent magnet, and the ends where the flux enters and leaves the iron are its poles.

$W = \eta B_{max}^{1.6}$

CHAPTER XIII

ELECTROMAGNETIC INDUCTION

164. Electromagnetic Induction.—When the current flowing through a circuit is started or stopped, or changed in any manner whatever, it is observed that other currents are set up in all of the other closed circuits which are near the first one. If some of these circuits are not closed, the tendency to produce a current is present just the same, but being open circuits no current results. In other words, an E.M.F. is induced in every conductor near a circuit in which the current is varying. If a current in the second circuit is varying, there will be a corresponding E.M.F. induced in the first circuit. This action is called mutual induction.

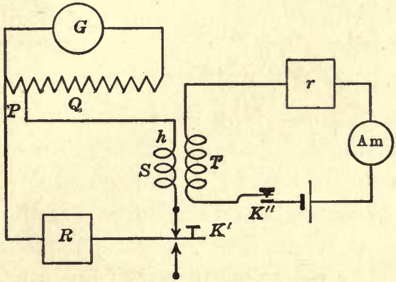

FIG. 78.—Mutual induction.

165. Laws of Mutual Induction.—A very satisfactory study of mutual induction can be made with a pair of coils and a sensitive ballistic galvanometer, connected as shown in Fig. 78. One of the coils is joined in series with a battery, an ammeter, a key, and a rheostat, *r*, for varying the current over a wide range. This is called the primary circuit, and whichever coil

is used in this circuit is called the primary coil. The other coil, called the secondary, is connected in series with the galvanometer and a resistance box. In order that the current may be varied without affecting the galvanometer when a reading is not desired, a switch or key, K' will be required in the secondary circuit.

It is also desirable to open the secondary circuit *before* making the reverse change in the primary current, in order to avoid deflecting the galvanometer to the other side of its resting point. There may be a shift of the zero, or resting, point of the galvanometer after it has been deflected in the opposite direction. In this case the first two or three readings in either direction should be discarded. In taking a series of readings it is best to keep the deflections in the same direction and never allow the galvanometer to swing across zero to the other side. If this precaution is taken the zero position should be quite constant.

166. The Effect of Varying the Primary Current.—The first part of this experiment is to study the relation between the change in the primary current and the resulting deflection of the galvanometer. The primary current is adjusted to such a value that closing K'', with K' closed of course, will give a fairly large deflection. Note the effect of opening K''; also the effect of closing K'' first, and then closing K'.

The actual value of the current has no effect upon the deflection, as this is the same when the current is changed from zero to one ampere as when the change is from four amperes to five amperes. If a primary current of one ampere is reversed, the change in the current is evidently two amperes, and the deflection will be twice as great as for either making or breaking the circuit.

Law I.—The quantity of electricity flowing through the secondary circuit depends directly upon the change produced in the primary current.

To investigate this relation, the rheostat r is adjusted to give the largest current that is to be used, and R and P are set so as

to make the corresponding fling of the galvanometer as large as can be conveniently measured. Starting with this current, the deflection is read when the circuit is closed. The reverse kick of the galvanometer is avoided by opening K' as soon as the reading is obtained, and then opening K''. This reading should be repeated to make sure of consistent results. The current should be kept flowing as little as possible to avoid heating the coils—especially changing the resistance of the secondary coil by warming it. The current is now reduced about 10 per cent. and another set of readings obtained, and so on until the current is reduced to zero.

Since the scale readings are proportional to tan 2θ, it may be necessary to correct them by the use of a calibration curve. The corrected readings are then plotted as ordinates against the corresponding changes in the primary current. This should be a straight line passing through the origin, and represented by the equation,

$$d = aI, \qquad (1)$$

where a is the slope of the curve. The value of a depends upon the number of turns of wire in the primary and secondary coils, and it also contains as one factor the reciprocal of the resistance of the secondary circuit.

167. The Effect of Varying the Secondary Resistance.—The second part of the experiment deals with the effect of changing the secondary circuit. Keeping the primary circuit constant, so that there will be the same change in the current each time the key is closed, a series of deflections are obtained by using different resistances in the secondary circuit. At each step two or three readings are taken in the same way as before. All of the resistance in this circuit should be measured under the conditions of the experiment, especially if the coil has been warmed appreciably by the current in the adjacent primary.

The galvanometer should not be joined directly in series with the rest of the circuit, for when the resistance of this circuit is varied it will alter the damping factor of the galvanometer, and

therefore its deflections will not be proportional to the quantities of electricity that are discharged through it. This trouble can be avoided by using a constant damping shunt consisting of two resistances, P and Q. P should be a few ohms, as many as necessary, and $P + Q$ sufficiently large to give critical damping to the galvanometer, that is, as large as possible and still keep the swing of the mirror aperiodic. A universal shunt of the proper resistance may conveniently replace $P + Q$.

The total resistance of the secondary circuit is

$$R' = R + h + p,$$

where h denotes the resistance of the secondary coil, p the combined resistance of the galvanometer and shunt, and R is the additional resistance in the box R.

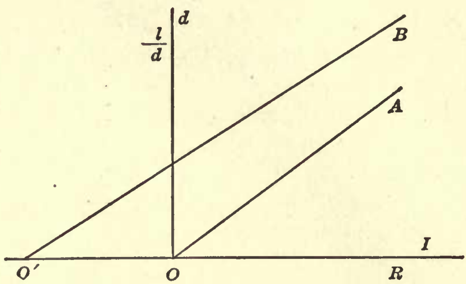

FIG. 79.—Laws of mutual induction.

It will be found that the deflections become larger as R' is made smaller by decreasing the resistance in R; and plotting the corrected deflections against $1/R'$ will give a straight line passing through the origin, as in the first part. Inasmuch, however, as R is the only observed part of R', and $h + p$ may be unknown, it is more satisfactory to plot $1/d$ as ordinates against the corresponding values of R. This will give a straight line as before, but not through the origin. Extending this back to the axis of R it will intersect it at a point $h + p$ ohms behind the origin of R: hence this point is the origin of R'.

The relation between d and R' is given then by the expression,

$$\frac{1}{d} = bR', \text{ or } d = \frac{1}{bR'} \tag{2}$$

where R' is the total resistance in the secondary circuit and b is the slope of the line $O'B$. The value of b depends upon the number of turns in each of the coils, and it also contains as one factor the constant value of the current which was made and broken in the primary circuit when the deflections were observed.

Law II.—The quantity of electricity flowing through the secondary circuit varies inversely as the resistance of the circuit.

From these two relations it is seen that the complete expression for the relation between the deflection, the change in the primary current, and the resistance of the secondary circuit is

$$d = f\,\frac{I}{R'} \tag{3}$$

where f is the proportionality factor. Evidently f/R' is the a of eq. (1), if R' is the constant resistance of the secondary circuit when curve A was determined. Likewise, fI is the b of Eq. (2) if I denotes the change that was made in the primary current when curve B was determined.

168. Meaning of Mutual Inductance.—The total quantity of electricity discharged through the galvanometer is, then,

$$Q = kd = kf\frac{I}{R'} \tag{4}$$

where k is the usual ballistic constant.

The ballistic galvanometer indicates the passage of a quantity of electricity through it, and from the nature of the circuit to which it is connected it is seen that this quantity does not come from the discharge of a condenser, but represents the

passage of a transient current. And furthermore, if there is a current in the secondary circuit there must be an E.M.F. causing this current.

On page 167 it was shown that an E.M.F. is induced in a wire which cuts across a magnetic flux. In the present case, as well as in the experiments of the preceding chapter, the wire is stationary while the flux cuts across it. When this wire is wound into a coil of n turns the flux cuts the wire in each turn, thus inducing an E.M.F. in each turn. It should therefore be possible to express the value of the E.M.F. in the secondary coil in a form similar to Eq. (6) in Art. 138. The total E.M.F. induced in the coil is, then,

$$e = n\frac{d\phi}{dt}, \tag{5}$$

where ϕ denotes the flux linked with the n turns of the coil. This is the E.M.F. causing the current through the galvanometer and the secondary circuit. In case the same flux does not pass through each turn of the coil the two factors, ϕ and n, of the "flux-turns" cannot be separated, but this quantity, ϕn, must then be considered as a summation extending to all the turns of the coil.

The two coils, T and S, Fig. 78, are similar to the primary and secondary windings on the iron ring Z, Fig. 77, except that now the magnetic circuit is wholly of air. The M.M.F. due to the current in T is $4\pi Ni'$, and this causes a flux,

$$\Phi = \frac{4\pi Ni'}{Reluctance}$$

through this coil and the surrounding air. The portion, ϕ, of this flux passing through the n turns of coil S depends upon the relative position of the two coils. Let p denote this fraction. Then $\phi = p\Phi$, and the E.M.F. induced in this coil by the change in the primary current is,

$$e = \frac{d(p\Phi n)}{dt} = pn\frac{d}{dt}\left(\frac{4\pi Ni'}{Rel.}\right) = \frac{4\pi pNn}{Reluctance}\,\frac{di'}{dt}. \tag{6}$$

Writing a single symbol for these various constants,

$$e = M\frac{di'}{dt}, \tag{7}$$

where the coefficient M is called the coefficient of mutual induction, or simply the *mutual inductance* of the pair of coils.

Definition.—One henry of mutual inductance is the inductance between two circuits when an E.M.F. of one volt is produced in one of them when the inducing current in the other changes at the rate of one ampere per second.

Strictly speaking, eq. (7) should be written $-e$, for if attention is paid to the direction of e when the current is increased, that is when dI is positive, it is found to be directed round the coil in the opposite sense to the current, I. If we are looking only for the numerical values of e and M the sign does not matter, but in case the direction of the induced current is considered it is necessary to write,

$$-e = M\frac{di'}{dt} \tag{7'}$$

168A. Value of the Mutual Inductance.—When a varying current flows through a circuit which is wound into a coil, like the primary and secondary coils in the present case, it is necessary to modify the statement of Ohm's law as given for steady currents. The E.M.F. necessary to maintain a varying current is,

$$e = Ri + L\,\frac{di}{dt},[1]$$

where Ri is the E.M.F. necessary to maintain the current through the resistance R, and $L\, di/dt$ is the E.M.F. required to make the current change by an amount di in the time dt, because of the self inductance, L, of the circuit.

[1] It is necessary to add this term in order to keep the equation correct. It drops out upon integration and does not affect the final result. The full meaning of this term is explained in the proper place a few pages below. (See eq. (5), Article 172.)

Integrating this equation with respect to the time,

$$\int_t^{t'} edt = \int_t^{t'} Ridt + \int_t^{t'} Ldi = R \int_{q=0}^{q=Q} dq + L \int_{i=0}^{i=0} di = RQ \quad (5)$$

where the integration is extended from the time, t, before the primary current begins to flow till the time, t', when the primary current has reached its steady value, I.

But from (7),

$$\int_t^{t'} edt = \int_0^{I} Mdi' = MI,$$

and therefore,

$$MI = RQ, \tag{4'}$$

or, solving for the value of the mutual inductance,

$$M = \frac{QR}{I} \tag{8}$$

If I is expressed in amperes, R in ohms, and Q in coulombs, then M will be given in henries.

169. Calculation of the Value of M.—Returning now to the consideration of the first part of this experiment, let R' denote the fixed value of the resistance of the entire secondary circuit. Then from (8),

$$M = R' \frac{Q}{I} = R' \frac{kd}{I} = R'ka, \tag{9}$$

where each symbol has the meaning that has been assigned above. The value of k can be determined as shown below, while R' and a are obtained from the curves.

From the second part of the experiment, in which the primary current was kept constant at I' amperes,

$$M = \frac{Q\,R'}{I'} = \frac{kdR'}{I'} = \frac{k}{bI'}. \tag{10}$$

The constant b is the slope of the curve $O'B$, and k is determined as follows.

170. To Determine the Constant of the Ballistic Galvanometer.—When the constant of a ballistic galvanometer is

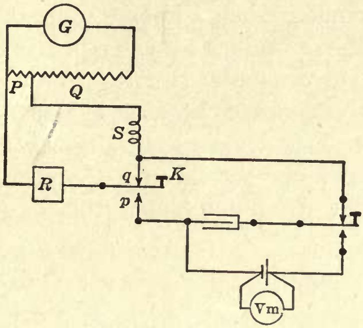

FIG. 80.—To determine the constant of *G*.

determined by the usual method with a condenser and standard cell (see Article 32) the galvanometer is used on open circuit. When it is used on a closed circuit, as in the case above, the damping is very much greater, and the same quantity of electricity discharged through the galvanometer will produce a much smaller deflection. Therefore the constant should be determined with the galvanometer under the conditions in which it has been used. This requires that the secondary circuit be opened long enough to discharge the condenser through it, and then immediately closed again. This can be done by using two keys, as shown in Fig. 80, or better by a double testing key in which all the contacts are made during a single motion of the hand.

This method for determining the constant of the galvanometer is not recommended where much accuracy is required and it should be resorted to only when no other method is available. In theory it is simple, but owing to the increased damping of the galvanometer when connected to a closed circuit it is difficult to determine just what value of d should be used for computing k. The following method is more satisfactory and gives more accurate results.

171. Constant of a Ballistic Galvanometer by Means of a Coil of known Mutual Inductance.—The foregoing experiment furnishes a method for obtaining the constant of a ballistic galvanometer when a known mutual inductance is at hand. For let the secondary of such a pair of coils be included as a part of the galvanometer circuit. When not in use it will not affect the galvanometer except as so much resistance in the circuit, and when it is desired to find the constant k, it is only necessary to pass a current through the primary and observe the galvanometer deflection when this current is made or broken.

From eq. (9) we have,

$$MI = R'kd, \qquad \text{or,} \qquad k = \frac{MI}{R'd}.$$

This use of a known mutual inductance is especially useful in cases where it is desired to obtain the constant of the galvanometer without opening the galvanometer circuit. In fact it would be useful in the above experiment if the use of a mutual inductance is thoroughly understood by the student.

171A. Secondary Definition of Mutual Inductance.—In the experiments of Chapter XII, a different constant was required, viz., the magnetic ballistic constant, c, whch was used in the relation,

$$\phi n = cd \tag{1}$$

where d is the deflection due to a change of ϕn flux-turns through the test coil.

The E.M.F. induced in this case was, (see Article 138).

$$e = -n\frac{d\phi}{dt} \tag{2}$$

In the case above (see Article 168), the E.M.F. induced in the secondary coil was,

$$e = -M\frac{dI}{dt} \tag{3}$$

where I was the prımary current.

Equating (2) and (3), and integrating, gives,

$$\phi n = MI \tag{4}$$

Therefore, when a current I is flowing through the primary of a pair of coils whose mutual induction is M, there are ϕn flux-turns through the secondary, and it is the changing of this flux that causes the induced E.M.F. in the secondary. If the secondary is so arranged that all of the flux passes through each of its n turns, then ϕ is the value of the actual flux. In case some of the flux does not pass through all of the turns, this product is to be considered as a summation, counting the actual flux each time it passes through a turn of the secondary. In either case the value of ϕn is determined by measuring M, and therefore the question whether all the flux passes through all the turns of the secondary is of no moment.

Evidently all of the quantities in Eq. (4) must be measured in C.G.S. units.

172. Meaning of Self Inductance.—It is but a step from considering the action of a current on an adjacent circuit to the case of action upon the same circuit. If the second circuit above were included as a part of the primary circuit then the induced E.M.F. would be in the same circuit as the inducing current, and in a direction opposed to it. Furthermore, this same kind of action would appear in every turn of the primary coil, the current in each turn inducing an E.M.F. in each of the other turns. The total E.M.F. induced in the circuit by the inducing current is,

$$e = -L\frac{di}{dt}.$$

where the $-$ sign means the same as before and L is a constant depending upon the number of turns and the dimensions of the circuit. It is called the *Self Inductance* of the circuit and is measured in henrys.

Definition.—One henry of self inductance is the inductance in a circuit when an electromotive force of one volt is produced by the current changing at the rate of one ampere per second.

173. Starting and Stopping a Current.—When an E.M.F., E., is applied to a circuit, the current starts at the value zero and increases to its final steady value. While the current is thus increasing there is induced in the same wire an E.M.F. which is opposed to the current. The value of this E.M.F. is

$$e = -L\frac{di}{dt}.$$

The total E.M.F. in the circuit at any instant is, therefore,

$$E + e = E - L\frac{di}{dt} \tag{2}$$

and by Ohm's law the current at the same instant will be,

$$i = \frac{E - L\dfrac{di}{dt}}{R} \tag{3}$$

At the start, when the current is changing most rapidly, the second term in the numerator is nearly equal to E. As the current approaches its final value it changes more slowly, the counter E.M.F. becomes small, and finally the steady value of the current is

$$I = \frac{E}{R}. \tag{4}$$

Equation (3) is often written in the form

$$E = Ri + L\frac{di}{dt} \tag{5}$$

which shows that while a part, Ri, of the applied E.M.F. is effective in maintaining the current i, another part, $+ L\frac{di}{dt}$, is required to make the current increase. This equation is very important in all discussion of varying currents.

174. Dying Away of a Current.—Suppose that a steady current has been flowing in a circuit when suddenly the battery, or other source of E.M.F. is removed without breaking the circuit or in any way changing its resistance or inductance. Eq. (5) now becomes

$$0 = Ri + L\frac{di}{dt} \tag{6}$$

and the current thus left to itself dies away to zero, as is shown by the following relations.

Rewriting (6) to separate the variables,

$$\frac{di}{i} = -\frac{R}{L}dt. \tag{7}$$

The integral of this is

$$\log i = -\frac{Rt}{L} + c \tag{8}$$

or in the exponential form,

$$i = C\epsilon^{-\frac{Rt}{L}} \tag{9}$$

The value of the constant of integration, C, is given by the fact that at the start, when $t = 0$, the current had the value I. Putting these values in (9) gives, $I = C$. Therefore (9) becomes

$$i = I\epsilon^{-\frac{Rt}{L}} \tag{10}$$

From this equation it appears that it is the self inductance that prevents the current from falling to zero immediately, and the greater the self inductance the more slowly will the current die away. If it is desired to state how rapidly the current decreases it is necessary to state how long it takes for the current to fall to half value—or to some other definite fraction of its original amount—since the value of i from (10) will reach zero only after an infinite time.

When t has the value $\frac{L}{R}$, the current equals $\frac{I}{\epsilon}$, or $\frac{I}{2.72}$, and this interval in which the current falls to 0.368 of its original value is called the *time constant* of the circuit.

175. Beginning of a Current.—When a circuit containing an E.M.F. is closed the current rises from zero to its final value at a rate depending upon the self inductance in the circuit. This rate of increase can be found from the equation

$$E = Ri + L\frac{di}{dt}.$$

Separating the variables gives this in the form

$$\frac{-\,di}{\frac{E}{R} - i} = -\frac{R}{L}dt,$$

the integral of which is

$$\log\left(\frac{E}{R} - i\right) = -\frac{Rt}{L} + c,$$

or,

$$i = \frac{E}{R} - C\epsilon^{-\frac{Rt}{L}}.$$

The constant of integration is determined by the condition that when $t = 0$, $i = 0$. Hence, $C = E/R = I$, where I is the final value of the current.

Therefore,

$$i = I - I\epsilon^{-\frac{Rt}{L}}.$$

Here also it is seen that it is the self inductance that keeps the current from rising suddenly to its full value as soon as the circuit is closed. The greater the self inductance in comparison with the resistance, the more slowly will the current rise, but it never quite reaches its maximum value. Therefore in comparing different circuits it is necessary to compare the periods taken for the currents to rise to half value—or to some other definite fraction of the final steady values. When $t = \frac{L}{R}$ the current lacks $\frac{I}{\epsilon}$ of its final value, and this period in which the current rises to 0.632 of its maximum value is called the *time constant* of the circuit.

Problem A.—Given a circuit in which the resistance is 10 ohms, and the self inductance is .01 henry, draw a curve showing the rise of the current for the first 0.004 of a second after applying a steady E.M.F. of 100 volts.

Problem B.—The above circuit is placed in parallel with a non-inductive resistance of 10 ohms, when the current in the main line is 20 amperes. Draw a curve showing how the current dies away in the parallel circuits when the main line switch is suddenly opened.

The logarithms which appear in the equations through the process of integration are, of course, not the common logarithms with the base 10, but are the natural logarithms with the base $\epsilon = 2.718 +$. Therefore in finding the time at which the current will have a given fraction of its maximum value, the following table will be useful.

Number	Natural logarithms.	
	Tabular value	Numerical value
0.9	9.895–10	−0.105
0.8	9.777–10	−0.223
0.7	9.643–10	−0.357
0.6	9.489–10	−0.511
0.5	9.307–10	−0.693
0.4	9.084–10	−0.916
0.3	8.796–19	−1.204
0.2	8.391–10	−1.609
0.1	7.697–10	−2.303
0.07	7.341–10	−2.659
0.04	6.781–10	−3.219
0.01	5.395–10	−4.605

CHAPTER XIV

MEASUREMENT OF SELF AND MUTUAL INDUCTANCE

176. Comparison of Two Self Inductances.—The self inductance of a coil can be measured by the bridge method, using variable currents. A Wheatstone bridge arrangement is set up as shown in Fig. 81, with the coil to be measured as one arm of the bridge and a variable standard of self inductance in the corresponding arm. The other two arms consist of two non-inductive resistance boxes which can be adjusted to balance the bridge. A galvanometer G and a battery B are connected in the usual way with the double key K', K''.

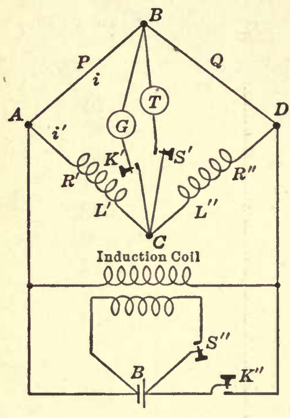

FIG. 81.—Comparison of two self inductances.

In order to use variable currents the secondary of an induction coil, or other source of alternating current, is connected to the bridge at the same points as the battery, without disturbing the latter connections. The primary of this induction coil is connected to the battery through the switch S''. The same battery can serve in both places since it will not be needed in one circuit while the other is being used. As an ordinary galvanometer is not deflected by an alternating current, a telephone receiver is connected between B and C by the switch S'. The two switches, S' and S'' may well be the two blades of a double pole switch which will close both circuits by a single motion.

The induction coil should be enclosed in a well padded box to reduce the noise as much as possible. By listening at the telephone receiver while the inductance of the standard is varied, the position for a minimum sound is readily determined. Before recording the readings the direct current balance should again be tried to make sure that the resistances have not changed while the second balance was being made. The contacts in the variable standard are not always constant, and perhaps other changes may occur. A Wheatstone bridge can be balanced for varying currents only when *both* the resistances and the inductances are adjusted to the proper values.

The relation between the inductances and resistances which will give this double balance may be found as follows: Applying Kirchhoff's law to the circuit $ABCA$ at any instant when the currents are i and i' gives

$$Pi - R'i' - L'\frac{di'}{dt} = 0 \tag{1}$$

similarly for $BDCB$,

$$Qi - R''i' - L''\frac{di'}{dt} = 0 \tag{2}$$

since there is no current from B to C as indicated by no sound in the telephone.

Transposing and dividing (1) by (2)

$$\frac{L'}{L''} = \frac{Pi - R'i'}{Qi - R''i'} = \frac{P(i - R'i'/P)}{Q(i - R''i'/Q)} = \frac{P}{Q}$$

since the resistances are adjusted for a direct current balance and therefore $R'/P = R''/Q$.

Thus for a balance with varying currents, the triple equation

$$\frac{L'}{L''} = \frac{R'}{R''} = \frac{P}{Q}$$

must be satisfied.

This means that each of the inductive arms of the bridge must be *equally inductive*, or in other words, the henrys per

ohm must be the same for each arm in order to give a balance with variable currents. All of this is on the supposition that L' is a constant, i.e., contains no iron or other magnetic substance. It is seen that the balance is independent of the particular way in which the currents are made to vary.

In case no balance can be obtained within the range of the standard, some non-inductive resistance may be added to the more inductive of the two inductive arms. And by varying this resistance several readings may be obtained on different parts of the scale.

An Inductance Bridge

An inductance bridge box is made by Leeds and Northrup, and in general appearance resembles the Wheatstone bridge

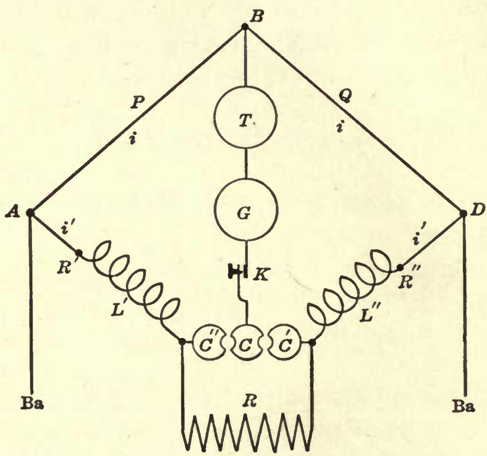

FIG. 81*a*.

box made by the same firm. The ratio arms are similar in the two boxes and each has a rheostat arm, but as the actual resistance in this arm is immaterial in the inductance bridge the fine adjustment is made by two small circular rheostats, each having many steps. The ratio arms correspond to P and Q

of Fig. 81, while the rheostat arm of variable resistance is connected between the two inductance coils as shown in Fig. 81a. By means of a plug key the galvanometer connection C may be joined to either C' or C'', thus putting the resistance R in series with either R' or R'' as desired. The resistance balance is easily made by varying R, after which the inductance balance is obtained by varying L'. The value of the unknown inductance is, then,

$$L'' = \frac{Q}{P}L'$$

where Q/P is an integer power of 10.

177. Comparison of a Mutual Inductance with a Self Inductance.—The preceding method gives also a very satisfactory way of measuring the mutual inductance between two coils. Call the self inductances of the coils L_m and L_n, and their mutual inductance M. The E.M.F. induced in these coils when connected in series and carrying a current i, is,

$$L_m\frac{di}{dt} + M\frac{di}{dt} + L_n\frac{di}{dt} + M\frac{di}{dt} = (L_m + L_n + 2M)\frac{di}{dt}.$$

But the coefficient of di/dt is by definition the inductance of the circuit, and if the two coils thus joined together in series were made one arm of the bridge shown in Fig. 81, the inductance measured by that method would be

$$L_s = L_m + L_n + 2M. \qquad (1)$$

If one of the coils were reversed, the E.M.F. induced in each coil by the mutual inductance of the other would be reversed also, and therefore the inductance measured by the bridge in this case would be,

$$L_r = L_m + L_n - 2M \qquad (2)$$

Subtracting (2) from (1) gives

$$L_s - L_r = 4M$$

Hence to determine the mutual inductance of a pair of coils it is only necessary to measure the self inductance of both together when joined in series—first with the coils direct, and again with one of the coils reversed. One-fourth of the difference between the two inductances gives the value of the mutual inductance.

178. Measurement of a Self Inductance by means of a Capacity. Maxwell's Method.—In this method the self inductance to be measured is placed in one arm of a Wheatstone bridge, the other arms of which should be as free from inductance as possible. By closing the keys in the usual order the bridge can be balanced for steady currents giving the relation

$$PS = QR \qquad (1)$$

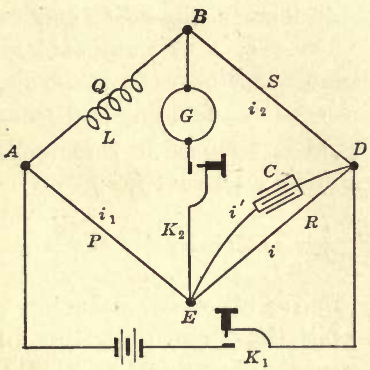

FIG. 82.—Comparison of capacity and self inductance.

But even though the bridge is balanced, if the galvanometer key is closed first there will be a large deflection upon closing the battery key. This is due to the current through the inductive branch ABD not being able to reach its full value as quickly as the current through the non-inductive branch AED, and therefore just at the start the fall of potential over S is not as great as that over R. This effect can be balanced by a suitable inductance in P, as was done in Article 176, or by placing a condenser in parallel with R as shown in Fig. 82. With the condenser so placed, a large part of the current through P, just at the start, will flow into the condenser and therefore the fall of potential over R will not be as large as it would be without the condenser. If the capacity of the condenser is of the proper amount this latter effect will just equal the effect of the coil in reducing the current through

S, and therefore the potentials of B and E will rise together. In this case there will be no deflection of the galvanometer even though its key is closed first.

In practice then the bridge is first balanced for steady currents. Then with K_2 closed the balance is tried again. If the value of C is not just right to give a balance with varying currents, and if it cannot be varied, its effect can be varied by changing R. This will necessitate a corresponding change in S and another trial with varying currents until this double balance has been obtained. A telephone in place of the galvanometer and alternating currents can be used equally well for the second balance. Or if the apparatus is at hand, it is convenient to use the alternating current generator with a commutator on the same shaft for rectifying the galvanometer current. The 140 volts generated by the machine is entirely too much to use directly on good resistance boxes. By means of a transformer the voltage can be reduced to twelve volts. The secondary can be connected in series with the battery of Fig. 82. When the machine is not running the direct current balance can be obtained the same as before. When the alternating current is used it will be superimposed on the battery, but will give a varying current all the same.

When no current flows from B to E we can write for any one instant,

$$Pi_1 = Qi_2 + L\frac{di_2}{dt} \tag{2}$$

and

$$Ri = Si_2 \tag{3}$$

where $i_1 = i + i' = i + \dfrac{dq}{dt} = i + \dfrac{d}{dt}(CRi) = i + CR\dfrac{di}{dt}$ (4)

since both the current i through R, and i' flowing into the condenser are added together to make the current i_1 through P.

Substituting in (2) the value of i_2 from (3) and i_1 from (4) gives

$$P\left(i + CR\frac{di}{dt}\right) = Q\frac{Ri}{S} + L\frac{R}{S}\frac{di}{dt}$$

or

$$(PS - QR)i + PCRS\frac{di}{dt} = LR\frac{di}{dt} \tag{5}$$

But because of the direct current balance (1) this reduces to

$$L = PSC. \tag{6}$$

and the bridge is balanced when S has the value required by (6) in addition to the requirement of (1).

PRACTICAL APPLICATIONS OF MAXWELL'S METHOD

179. Case A. Self Inductance Varied. *Calibration of a Variable Standard of Self Inductance.*—The double balance required in the previous experiment makes the method long and tedious. But if Q is a variable self inductance, some value of L can be found for each steady current balance. The method is therefore very useful in the calibration of a variable self inductance.

The operation of the calibration is simple. A direct current balance is obtained with values of P, S and C which will give a product equal to the value of L it is desired to calibrate. Then turning to alternating currents, the inductance is varied until a balance is obtained. If L' is the reading at this point, the correction to be applied is

$$c = PSC - L'.$$

A calibration curve should be plotted between L' as abscissæ and the corresponding values of c as ordinates.

180. Case B. Resistance Varied. *Inductance of a Single Coil.*—In case the self inductance to be measured is not variable, but, say, is a coil having a fixed and constant value for L, the balance can readily be obtained as follows: Let R and S, Fig. 82, be two similar decade resistance boxes so that the resistances of these two arms of the bridge can easily be kept equal to each other. Start with some convenient value, such as $R = S = 1000$ ohms.

In order to obtain the direct current balance it will now be necessary to make $P = Q$. Usually Q is not very large, and if P is a resistance box the balance can be obtained only to the nearest ohm. The final balance can be made by adjusting a low resistance rheostat in series with the coil and forming a part of the arm Q. The amount of resistance used from this rheostat, as well as the resistance of the coil itself, need not be known.

The battery and galvanometer are now replaced by an alternating E.M.F. and a current detector. By varying R and S together, thus keeping them equal, the alternating current balance is readily found and the value of S giving this balance is the one required in Eq. 6. If R and S are two similar decade boxes, as specified above, they can be kept equal and the balance found as easily as though a single resistance was varied. If several independent determinations of L are required, a small resistance may be added to Q each time and the measurements repeated.

If it is desired to measure L when the coil is carrying a certain constant current, P and R can be set equal and the balance obtained by adjusting S and Q in the same manner as before. By maintaining a constant alternating current voltage at AD, the current through Q will remain constant while P and R are being adjusted to give the alternating current balance. This arrangement is especially desirable when the coil has an iron core, or whenever the value of L depends upon the current through the coil.

181. Case C. Capacity Varied.—If the capacity of the condenser C can be changed by several small steps, it is often convenient to obtain the alternating current balance by varying the amount of capacity in parallel with the resistance R. Changing the capacity of C will have no effect on the resistances of the bridge, and therefore if once in balance it will not be disturbed by making the alternating current balance.

If the steps are too large to give an exact balance with varying currents, the two values nearest the balance can be

tried and the deflections noted. Then the value of the capacity which would give the exact balance can be determined by interpolation. This value of C is to be used in eq. 6 to compute the value of L.

182. Case D. Effect of Capacity Varied.—Sometimes it is necessary to measure the self inductance of a coil with a condenser of fixed capacity. In this case the effect of the capacity can be varied by putting the condenser in parallel with only a *part* of R. Let R consist of two resistance boxes, V and W, so arranged that while the resistance in either may be varied, yet the total resistance in both shall always be R. Let the condenser be placed in parallel with V only. The potential to which the condenser is now charged is only V/R of what it was before, and therefore the quantity it will receive is the same fraction of its previous charge. Furthermore, the effect of the condenser in reducing the initial fall of potential in this arm of the bridge will be only over the part V instead of over the whole of R as before. Hence again, the effect of the condenser is reduced to V/R of its former amount.

The effect, therefore, of a capacity C' when in parallel with V only is the same as that of a capacity

$$C = \frac{V^2}{R^2} C'$$

when in parallel with all of R. Putting this value of C into eq. 6 gives,

$$L = C' \frac{V^2}{R^2} PS$$

as the modified form of the equation.

183. Anderson's Method for Comparing a Self Inductance with a Capacity.—In Anderson's modification of Maxwell's method, the effect of the condenser is varied without disturbing the direct current balance. The setup differs from Maxwell's method by having a resistance, r, in series with the condenser, and then connecting the galvanometer at F, Fig. 83, instead of

at E. This makes no difference with the steady current balance, but with varying currents the effect of the condenser can be varied by changing the value of r. The alternating current generator with its commutator for the galvanometer circuit can be used to advantage.

When r has been adjusted to give no current through the

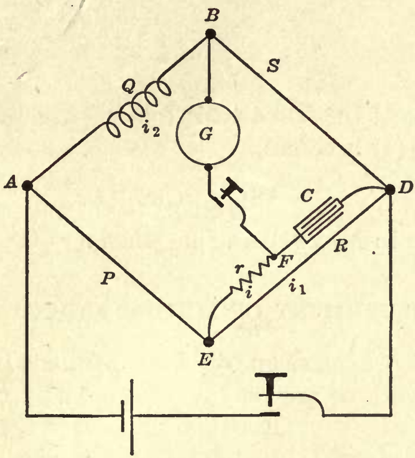

FIG. 83.—Comparison of capacity and self inductance.

galvanometer branch, the potentials of B and F must always remain equal. Then

$$P(i_1 + i) + ri = Qi_2 + L\frac{di_2}{dt} \tag{1}$$

where the currents are as shown in the figure. Similarly for the other branches

$$\frac{q}{C} = Si_2 \tag{2}$$

and

$$Ri_1 = ri + \frac{q}{C} \tag{3}$$

Substituting in (1) the values of i_1 and i_2 from (2) and (3) gives

$$\frac{P}{R}\left(ri + \frac{q}{C}\right) + (P + r)i = \frac{Q}{S}\frac{q}{C} + \frac{L}{SC}\frac{dq}{dt}$$

or,

$$\left(\frac{P}{R} - \frac{Q}{S}\right)\frac{q}{C} + \left(\frac{Pr}{R} + P + r\right)i = \frac{L}{SC}i \tag{4}$$

since

$$\frac{dq}{dt} = i$$

Making use of the fact that $PS = QR$ from the steady current balance, (4) becomes,

$$L = PSC + rC(S + Q), \tag{5}$$

which reduces to Maxwell's value when $r = 0$.

MEASUREMENT OF MUTUAL INDUCTANCE

184. Direct Comparison of Two Mutual Inductances.—Mutual inductances are readily measured by comparing the E.M.F's. induced in their secondaries. If the current through the primary coils is direct the induced E.M.F. will appear at the make and break of the circuit; while if alternating current is used there will be an alternating E.M.F. in the secondary.

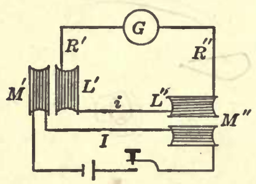

FIG. 84.—Comparison of two equal mutual inductances.

In the direct comparison of two mutual inductances the two primaries are joined in series and connected to the source of current, preferably a low-voltage alternating circuit. The two secondaries are also joined in series, with the two induced E.M.F's. opposed to each other. This method requires a variable standard of mutual inductance and a telephone, or a galvanometer and a rotating commutator, to indicate zero current in the secondary circuit. By turning the movable

coil of the standard until the E.M.F. induced in it is just equal and opposite to that induced in the secondary of the other pair of coils, as shown by zero deflection of the galvanometer, the value of the mutual inductance can be read directly from the standard. This is evident from the following considerations.

Writing Kirchhoff's law for the complete secondary circuit when not in balance gives, at any instant, t,

$$M'\frac{dI}{dt} - R'i - L'\frac{di}{dt} - Gi - L_g\frac{di}{dt} - M''\frac{dI}{dt} - R''i - L''\frac{di}{dt} = 0,$$

where G and L_g refer to the galvanometer.

For a balance $i = 0$, and is constant.

Therefore,

$$M' = M''$$

It thus appears that only mutual inductances of the same value as the standard, that is, up to 10 millihenrys, can be measured by this method. However the range can be extended by adding to the variable standard another mutual inductance of fixed value. This is done by joining the primaries in series and also the secondaries, thus adding together the E.M.F.'s induced in the two secondaries.

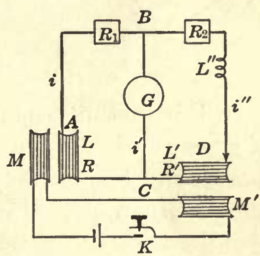

FIG. 85.—Comparison of two mutual inductances.

185. Comparison of Two Unequal Mutual Inductances.—Let M be a pair of coils whose mutual inductance is known and M' another pair whose mutual inductance is desired. The primaries of the two coils are joined in series with a battery and key, the coils themselves being placed as far apart as possible and at right angles to each other so that each secondary will be influenced only by its own primary. The two secondaries are also joined in series with two resistance boxes, and a galvanometer is connected across from B to C. This is not a Wheatstone bridge arrangement for the two

electromotive forces in the two secondaries act together and the current flows through the four arms in series.

By properly adjusting R_1 and R_2 the galvanometer will give no deflection when K is opened or closed. Writing out Kirchhoff's law for the circuit ABC gives, at the instant t,

$$M\frac{dI}{dt} - Ri - L\frac{di}{dt} - R_1 i - Gi' - L_g\frac{di'}{dt} = 0 \qquad (a)$$

and for BDC

$$M'\frac{dI}{dt} - R'i'' - L'\frac{di''}{dt} + Gi' + L_g\frac{di'}{dt} - R_2 i'' - L''\frac{di''}{dt} = 0$$

where L_g denotes the inductance of the galvanometer circuit.

Integrating each of these equations from the time when the key is first closed and $I = 0$, to the time when the primary current has reached its steady value, $I = I_0$, gives,

$$(R + R_1)q = MI_0, \text{ and } (R' + R_2)q'' = M'I_0,$$

if q', the integrated current through the galvanometer, is zero. Dividing one equation by the other gives

$$\frac{M}{M'} = \frac{R + R_1}{R' + R_2}$$

In this method the galvanometer current may be zero, but in general it is the sum of two transient currents each of which has an effect upon the galvanometer even when following one another in a short interval of time. This produces an unsteadiness of the galvanometer and renders an exact setting difficult, if not impossible. In order that no current should pass through the galvanometer it is necessary that the potential difference between B and C shall remain zero for each instant while the primary current is changing, and this requires that the self inductance of each branch shall be proportional to the E.M.F. induced in that part of the circuit.

Usually this is not the case, but it is not difficult to add some self inductance, L'', in the part of the circuit that is deficient

and thus fulfill this condition. The galvanometer will then indicate a much closer balance, or it may be replaced by a telephone and an alternating current used in the primaries.

If the apparatus is at hand, the telephone may well be replaced by a galvanometer and rotating commutator which will reverse the galvanometer terminals as often as the alternating current is reversed. This is more sensitive than the telephone but it can be used only where the instantaneous galvanometer current is zero for a balance. In other words this arrangement indicates zero current. It could not be used in the first arrangement where a balance was indicated by zero value of the integrated current.

185a. Comparison of a Large Mutual Inductance with a Small One.—When the two mutual inductances differ very widely in value it is not always possible or convenient to determine their ratio directly as in the previous method. Two methods of reducing the effect of the larger inductance are available, and both are here outlined.

First.—Refering to Fig. 84, in the method for comparing two equal mutual inductances the E.M.F's. induced in the two secondaries were made equal by varying one of the inductances, When this cannot be done, the balance can still be obtained by reducing the primary current in the larger mutual inductance by means of a shunt

Then,

$$MI = M'I',$$

and,

$$\frac{M}{M'} = \frac{I'}{I} = \frac{P}{P+Q}$$

where Q and P are resistances of the primary coil and its shunt respectively See Fig. 86.

Second.—By shunting the secondary of the stronger mutual inductance, as indicated in Fig. 86, the effect of its larger

E.M.F. can be balanced against the smaller E.M.F. of the other coil.

Writing out Kirchoff's law for the galvanometer circuit gives.

$$M\frac{dI}{dt} - Ri - L\frac{di}{dt} - R_g i - L_g\frac{di}{dt} - S(i + i') = 0.$$

The corresponding equation for the circuit consisting of the other coil and its shunt, is,

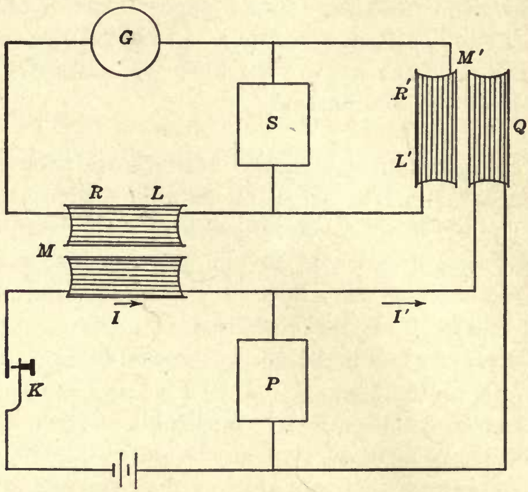

FIG. 86.—Comparison of two very unequal mutual inductances.

$$M'\frac{dI'}{dt} - R'i' - L'\frac{di'}{dt} - S(i + i') = 0.$$

Integrating these equations from the time that the primary circuit is closed until the primary current has reached its final steady value, gives,

$$MI - (R + R_g)\int idt - 0 - S\left[\int idt + \int i'dt\right] = C = 0.$$

and

$$M'I' - R'\int i'dt - S\left[\int idt + \int i'dt\right] = C' = 0.$$

The constants of integration are zero for the case when the final steady value of the primary current is zero, and hence they must always be zero.

If the galvanometer deflection is zero it shows that the total quantity, $\int idt$, that passed through the galvanometer is zero. Putting this value into the equations above, they become,

$$MI = S\int i'dt$$

and

$$M'I' = (R' + S)\int i'dt$$

Dividing the first by the second gives,

$$\frac{M}{M} = \frac{S}{R' + S}\,\frac{I'}{I} = \frac{S}{R' + S}\,\frac{P}{P + Q}$$

where the relation between the final steady values of the currents I and I' in the primary circuit is given by the usual law of shunts.

This is the relation between the two mutual inductances when both shunts are used. If either one is omitted ($= \infty$) the corresponding factor becomes unity.

186. Measurement of a Mutual Inductance in Terms of a known Capacity. Carey Foster's Method.—In this arrangement, shown in Fig. 87, one pair of coils is replaced by a condenser, and the transient current from the secondary coil S is balanced against the current that is charging the condenser C.

If the condenser were removed there would be a current from S through R' and the galvanometer each time the key in the primary circuit is closed, and a current in the opposite direction when the key is opened. With the condenser in place this current arrives at C just in time to charge the condenser, or to help charge it. The final charge in the condenser is

$$Q = C \times V = CRI \tag{1}$$

where I is the final steady current through R. The amount of this charge is the same whether there is a current through the galvanometer or not. By adjusting the resistance in R', the current through it can be made too small, too large, or just sufficient to supply the charge in the condenser, the rest of the charge coming directly through the galvanometer. When there is no deflection of the galvanometer the adjustment is complete, and the arrangement is said to be balanced.

Writing out Kirchhoff's law for the circuit through $SR'G$, gives,

$$M\frac{dI}{dt} - L\frac{di}{dt} - (S + R')i + Gi' + L_g\frac{di'}{dt} = 0. \qquad (2)$$

Integrating this between the limits of time t' just before K is closed, to t'', when the primary current has reached its steady value I, gives,

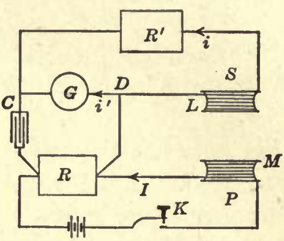

FIG. 87.—Comparison of a capacity and a mutual inductance.

$$MI - (S + R')Q' + Gq = 0, \qquad (3)$$

where Q' is the total quantity passing through R', and q is the quantity through the galvanometer. Then, $Q' + q = Q$, the total charge in the condenser. But if the galvanometer deflection is zero, then $q = 0$, and

$$MI = (S + R')Q = (S + R')\,CRI$$

from (1). Hence

$$M = (S + R')\,CR$$

It is to be noted that the galvanometer current is not required to be zero for a balance, and in fact it usually is not zero for each instant from t' to t''. Zero deflection merely indicates that the algebraic sum of the *quantities* passing through the galvanometer is zero. Nevertheless, the more nearly the galvanometer currents are to zero at each instant the more steady will be the zero deflection at the balance.

CHAPTER XV

ALTERNATING CURRENTS

187. An Alternating Current is the same as any other electric current, except that it flows in one direction for only a very short time; it then reverses and flows in the other direction for an equally short time. In ordinary lighting circuits there are from 100 to 300 such reversals each second. In some other cases there may be many millions of reversals each second. While the current is flowing in one direction it is the same as any other current of the same number of amperes. The only peculiarity of an alternating current is that it is continually being made to change. And just as a material body cannot change its velocity from one direction to the opposite without first slowing down to zero and then starting up in the other direction, so the current cannot instantly change from its full value in one direction to the full value in the other, but it requires some time to die down to zero and then to build up in the opposite direction. It does not have time to build up very far before it must begin to decrease again, so there is never a time when the current is not changing in amount. In fact, the value of the current as it changes from one direction to the other and back again goes through the same variations as the velocity of a pendulum bob when swinging to and fro.

188. Tracing Alternating Current and E.M.F. Curves.—In Chapter VII there were given some methods for measuring the current flowing through a circuit. The same arrangements can be used to measure the value of an alternating current. Since the balance point would vary rapidly up and down the slide wire of the potentiometer it will be necessary to add some mechanical device that will close the galvanometer circuit

for only an instant at the particular time when the current has the value that it is desired to measure. As the current will have the same value sixty (say) times in each second, the galvanometer circuit can be closed sixty times each second, and this is often enough to produce a steady deflection of the galvanometer when the potentiometer is not balanced. Thus the actual setting is made as easily as when the current is steady.

Let AD represent the slide wire potentiometer, S the resistance through which the alternating current is flowing, and M the instantaneous contact maker which closes the galvanometer

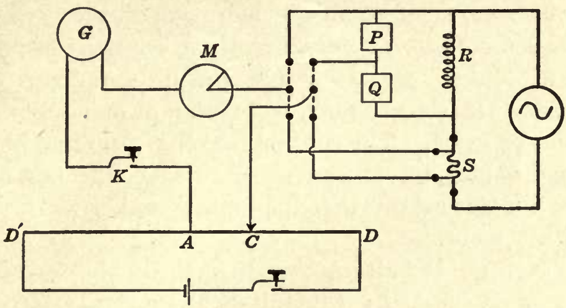

FIG. 88.—Potentiometer for tracing *a.c.* curves.

circuit for a very short time once in each cycle. Let i denote the value of the alternating current at the instant the galvanometer circuit is closed by M. The fall of potential over S is then Si, and if C is moved so that the fall of potential over AC is the same as Si there will be no deflection of the galvanometer. Therefore the distance AC is proportional to the current i.

Now let the contact maker M be turned a few degrees so as to close the circuit just a little later than before. The value of the current at this point will be somewhat different, and its value can be found by moving C along till a balance is again obtained. In this way the values of the current for the com-

plete cycle can be determined, and a curve plotted showing how the current varies with the time.

When the current reverses and the fall of potential over S is in the other direction, C must be moved to the other side of A to find a balance. Therefore AD' is merely a second slide wire potentiometer on which all negative values of the current can be measured. In this manner the values of the current at various instants corresponding to the different settings of M can be measured. These should extend far enough to complete at least one cycle, that is, until the readings begin to repeat themselves. If the dynamo that is generating the current has two poles there will be one cycle for each revolution of the armature. If there are more poles, there will be as many cycles in each revolution as there are pairs of poles.

It is absolutely necessary that the instantaneous contact maker, M, closes the galvanometer circuit at precisely the same point in each cycle. The contact maker is therefore placed on the dynamo shaft, but there is no electrical connection between the two; but this insures that the contact maker will keep rigid step with the current.

The curve for the alternating E.M.F. can be traced in the same manner. Usually the full E.M.F. is too large to be measured directly by the potentiometer, but by the fall of potential method, as used in the method for calibrating a high reading voltmeter, any small portion of this E.M.F. can be obtained. It is then only a matter of increasing the scale to get the curve of the full E.M.F. When the current and E.M.F. curves are both drawn on the same sheet the phase relation between them is clearly shown.

After tracing the curves for a current flowing through a non-inductive resistance it will be interesting to do the same for a coil having considerable inductance. This should show the effect of inductance in making the current "lag behind the E.M.F." If desired the experiment can be further varied by connecting a condenser in the circuit and finding the effect it has upon the current and E.M.F. curves.

189. Measurement of an Alternating Current.—The usual alternating current follows closely the sine law, and its value at any instant is given by the equation,

$$i = I \sin \omega t$$

where I and ω are constants.[1]

Instruments, however, are seldom made to give the instantaneous value of the current, but they record some kind of an average value. Evidently the arithmetical average of the values of a current which is negative as much as it is positive, would be zero and therefore such an instrument as a galvanometer, or an ordinary ammeter, would be useless for the measurement of an alternating current. But an electrodynamometer, or the Kelvin balance, measures the current equally well whichever way the current flows through it.

189A. Instantaneous Values of the Current.—When an alternating current flows through a circuit there are induced other E.M.F's. besides that impressed by the dynamo; and these extra E.M.F's. often have considerable influence in determining what the resulting current shall be. As we have seen, Article 172, because of the fact that the current is changing at the rate of $\frac{di}{dt}$ amperes per second, there will be induced in the circuit an E.M.F. of $-L\frac{di}{dt}$ volts. In order to maintain the current i, the dynamo must furnish not only the E.M.F. Ri, required by Ohm's law, but in addition must supply an E.M.F. sufficient to counter balance, at each instant, this induced E.M.F. That is, the dynamo must supply an E.M.F. whose value at any instant is

$$e = Ri + L\frac{di}{dt}$$

This is the general equation for any current varying in any

[1] In case the current can not be represented by a single term it can always be expressed by a series of such terms.

manner whatsoever. Solving this equation for the current gives,

$$i = \frac{e - L\dfrac{di}{dt}}{R}$$

which shows that the instantaneous value of the current is given by the usual form of Ohm's law—taking into account all of the E.M.F's. in the circuit at that instant.

190. Definition of an Ampere of Alternating Current.—It is evident that an alternating current has no steady value, and to speak of one ampere of alternating current is meaningless without some definite convention or definition. An alternating current will heat a wire, or the filament of an electric lamp, through which it passes, the heat produced each second being given by the formula, Ri^2. Although this value fluctuates widely the light from the filament appears continuous. By convention then, one ampere of alternating current is that amount which will bring a lamp to the same brightness as one ampere of steady direct current.

In a hot wire ammeter the current is measured by the increase in length of a fine wire due to the heat produced in it by the passage of the alternating current.

If an electrodynamometer has been calibrated for direct current it may be used to measure alternating current, and that amount of current which will give the same deflection as one ampere of direct current is called one ampere of alternating current.

191. Average Value of a Sine Current.—To find the average ordinate of the sine curve it is only necessary to determine the area included between the curve and the axis of abscissæ, and divide this by the length of the base. Then

$$\text{Average ordinate} = \frac{\text{area}}{\text{base}} = \int_0^{\pi} \frac{y\ dx}{\pi}$$

But for a sine curve,

$$y = a \sin x$$

Hence,

$$\text{Average ordinate} = \frac{a}{\pi}\int_0^{\pi} \sin x\, dx = -\frac{a}{\pi}\Big[_0^{\pi} \cos x = \frac{2a}{\pi}$$

$$= 0.6369 \text{ of maximum ordinate.}$$

192. Mean Square Value of a Sine Current.—Inasmuch as the heating and power effects of an electric current, as well as the dynamometer readings, depend upon the square of the values of the current, it is more useful to know the average value of the square of the current than merely its average value. This can be found in the same way as before. Since the squares of negative quantities are positive we are not limited to half a period, but can extend the integration over the whole period.

$$\text{Mean square of } y = \int_0^{2\pi} \frac{y^2\, dx}{2\pi} = \frac{a^2}{2\pi}\int_0^{2\pi} \sin^2 x dx$$

But, $2 \sin^2 x = 1 - \cos 2x$, and

$$\int_0^{2\pi} \sin^2 x\, dx = \Big[_0^{2\pi} \left(\frac{x}{2} - \frac{\sin 2x}{4}\right) = \pi$$

Hence, mean square of $y = \frac{a^2}{2}$, and the square root of the mean square value of the current is 0.707 of maximum value.

Alternating current ammeters and voltmeters are calibrated to read this value of the current because the power expended in a circuit depends upon the square of the current. The heat produced in a hot wire ammeter is evidently proportional to the average square of the current, and the deflection of an electrodynamometer (Art. 78) likewise depends upon the average square of the current. Therefore when such instruments are calibrated to measure direct currents, if they are used

to measure alternating current they will indicate the square root of the instantaneous values of the current. This value is often called the "mean square" value for short. Sometimes it is called the "effective value."

193. To Find what E.M.F. is Required to Maintain a given Current.—Let an alternating current flowing in a circuit containing both resistance and self inductance, be given by the equation

$$i = I \sin \omega t \tag{1}$$

It is required to find a similar expression for the E.M.F. which will maintain this current.

The general equation is,

$$e = Ri + L\frac{di}{dt},\text{[1]}$$

where Ri is the E.M.F. required to maintain the current through the ohmic resistance, and $L\frac{di}{dt}$ is the E.M.F. to balance the induced E.M.F.

Substituting the above value of the current gives,

$$e = RI \sin \omega t + L\omega I \cos \omega t, \tag{2}$$
$$= RI \sin \omega t + L\omega I \sin (\omega t + 90°)$$
$$= k \sin (\omega t + a) \tag{3}$$

where k and a are new constants.

The values of k and a can be determined in terms or R, L and ω by expanding $\sin (\omega t + a)$ as follows:

$$k \sin (\omega t + a) = k \cos a \sin \omega t + k \sin a \cos \omega t \tag{4}$$

Comparing this with (2) it is seen that

$$k \cos a = RI, \quad \text{and} \quad k \sin a = L\omega I,$$

from which

$$k = I\sqrt{R^2 + L^2\omega^2} \quad \text{and} \quad a = \tan^{-1}\frac{L\omega}{R} \tag{5}$$

From Eq. 3 it is evident that the maximum value that e can ever have is k; or writing E for this maximum value of e,

$$E = I\sqrt{R^2 + L^2\omega^2}. \tag{6}$$

[1] See Article 173.

194. Graphical Solutions.—The term $RI \sin \omega t$ in (2) may be represented by a line OA, Fig. 89. If this line is considered as rotating counter clockwise with the angular velocity ω, its vertical projection at any instant, t, gives a length $RI \sin \omega t$.

In the same way the term $L\omega I \cos \omega t$ may be represented by a line OB, which must be drawn 90° ahead of OA since $\cos \omega t = \sin (\omega t + 90°)$. The value of e at any instant will be given by the sum of the vertical projections of these two lines or what is the same thing, by the vertical projection of OC, which is the geometrical sum of OA and OB.

From the figure it is evident that the length of OC is

$$I \sqrt{R^2 + L^2\omega^2}$$

and it is ahead of OA by the angle $a = \tan^{-1} \frac{L\omega}{R}$.

Problem.—Draw the curve representing the current as given in (1) for at least two cycles.

On the same axis draw the two components of the E.M.F. as given by (2) and by addition obtain the curve for e.

The following constants may be used, $R = 2$ ohms. $I = 1$ ampere, $\omega = 400$, $L = 0.01$ henry.

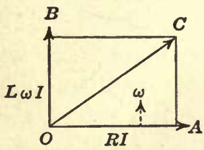

FIG. 89.—Addition of two harmonic quantities.

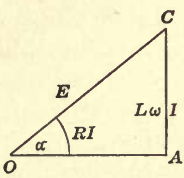

FIG. 90.—E.M.F. triangle.

Fig. 89 is called the parallelogram of E.M.F's. Since $OB = AC$, the triangle of E.M.F's., Fig. 90, shows the same relationships. The side OA represents the E.M.F. required to keep the current flowing through the resistance, R. This is called the effective component of the E.M.F. The side AC represents the E.M.F. required to just balance and oppose the induced E.M.F. due to the self inductance of the circuit. This is called the inductive E.M.F. sometimes wattless E.M.F. because

no energy is required to maintain it. The geometrical sum of these two gives the total E.M.F. that must be supplied to maintain the current, and it is called the impressed E.M.F.

The angle of lag of the current behind the impressed E.M.F. is shown by the angle a. This may have any value from 0° to 90° depending upon the amount of inductance in the circuit.

The Impedance Triangle.—If each side of Fig. 90 is divided by the current I, the result will be the triangle of resistances and impedance as shown in Fig. 91.

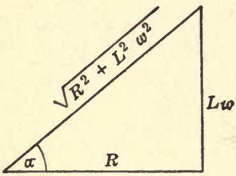

FIG. 91.—Impedance triangle.

The side R denotes the resistance, but this is not necessarily the same as the direct-current resistance. It is the factor which multiplied by the square of the current will give the amount of heat produced in the circuit.

The other side of the triangle, $L\omega$, is called the reactance. It is also measured in ohms, but there is no loss of energy or production of heat because of it. The hypotenuse is called the impedance.

195. Mean Square Values.—In all these diagrams I denotes the maximum value of the current and when so used the result obtained for E will be the maximum value of the E.M.F. If the ammeter reading is used for I, the same construction can be used and the result will be the voltmeter reading value of E. But in this case the diagram no longer has the significance that was given it in Fig. 89, and it becomes merely a graphical construction to reach a desired result.

196. Measurement of Impedance by Ammeter and Voltmeter.—From what has just been said it will be seen that the impedance of a circuit is merely the ratio of the impressed E.M.F. to the resulting current. To measure it, therefore, it is only necessary to measure the voltage and current in precisely the same way as the direct-current resistance is measured by an ammeter and a voltmeter. The ratio gives the impedance or,

$$\frac{E}{I} = \text{Impedance} = \sqrt{R^2 + L^2\omega^2}$$

197. Self Inductance by the Impedance Method.—If the values of R and ω are known, this method gives a means for computing the self inductance, L, of the circuit. If the current passes through n cycles per second, then $\omega = 2\pi n$.

The direct-current value may be used for R if the wire is not too large, and if there are no closed circuits, or masses of metal or iron in the vicinity.

In case heat is produced, or energy is otherwise expended, outside of R it will be necessary to determine one other quantity before the impedence triangle can be drawn. This may take the form, either of finding the angle of lag of the current behind the impressed E.M.F., or of determining the equivalent resistance of the circuit—that is, the non-inductive resistance in which the same amount of energy would be expended.

198. Impedance and Angle of Lag by the Three Voltmeter Method.—In this method a non-inductive resistance, capable

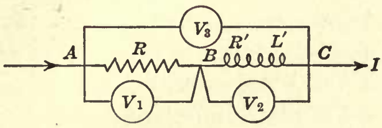

FIG. 92.—Three voltmeter method.

of carrying the current is placed in series with the impedance to be measured. Three voltmeter readings are taken, as nearly simultaneously as possible, to measure the voltage across each the resistance and the impedance and across both together. It is best to use three voltmeters as shown in Fig. 92, but if only one is available it may be transferred quickly from one position to another. If the voltmeter shunts an appreciable current from the main circuit, two equivalent resistances should occupy the places of the missing voltmeters.

The voltmeter is readily transfered to the various positions by means of two double throw switches, S and T, as shown in Fig. 93, where Vm denotes the voltmeter, and U and W are

resistances, each equal to to the resistance of the voltmeter. With both switches thrown to the right the voltmeter is placed across AC, and measures the total voltage over both the impedance and the non-inductive resistance. When both switches are thrown to the left the voltmeter is across BC, and measures the voltage over the impedance alone. When the switch S is thrown to the left and the switch T is to the right, the voltmeter is across AB, and measures the voltage over the resistance R, only. The resistances V and W are simultaneoulsy transferred to the positions not occupied by the voltmeter.

Referring to Fig. 90 it will be seen that E_2, the reading of the

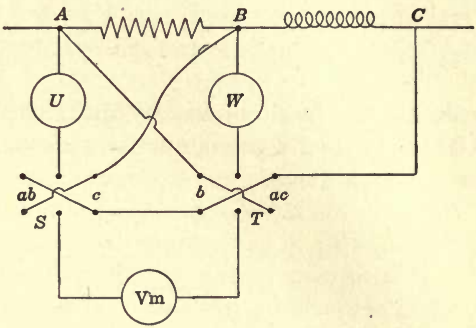

FIG. 93.—Switches for putting the voltmeter in three places.

voltmeter across BC, is the E.M.F. impressed upon the coil, and it would be the hypotenuse of the corresponding E.M.F. triangle if the other elements were known. In the same way E_3 is the hypotenuse of the E.M.F. triangle for the entire circuit AC, while for the part AB in which there is no inductance the hypotenuse coincides with the base of the triangle, and is measured by E_1.

Combining these three E.M.F's. gives the triangle ABC, Fig. 94. Extending the side AB until it meets the perpendicular from C gives the complete E.M.F. triangle for the entire circuit AC. The angle DBC gives the lag of the current behind E_2, and therefore BDC is the E.M.F. triangle for the coil.

If the current is also known, either by direct measurement or by computation if R is known, the impedance of the coil is given by the relation

$$\text{Impedance} = \frac{E_2}{I}.$$

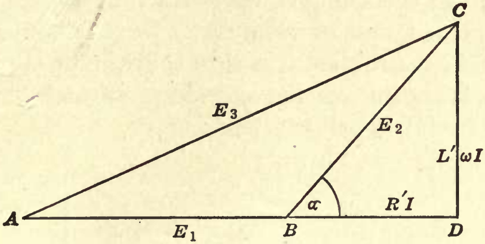

FIG. 94.—Determination of the angle of lag.

199. Determination of Equivalent Resistance.—Knowing the value of one side of the impedance triangle and the angle a, the other sides are readily constructed. The base of this triangle, R, gives the value of the *equivalent resistance* of the coil. The effect of a solid iron core is to greatly increase this equivalent resistance over the ohmic resistance of the coil as measured by direct-current methods. The inductance is likewise increased. If the core consists of a bundle of fine iron wire the increase of the resistance is less, while the inductance is greater than with the solid core.

200. Inductive Circuits in Series.—When two inductive circuits are joined in series the same current must, of course, flow through them both. But in general the E.M.F. over one will not be in phase with that over the other and therefore the total E.M.F. required to maintain the current will be less than the sum of the two parts. This is readily seen from the figure. $A'B'C'$ represents the two inductive circuits in series, and the diagram ABC shows the E.M.F. triangles for each part, and for the whole circuit. The triangle ANB is the E.M.F. triangle for the portion AB, and corresponds to Fig. 90 for the first part of the circuit.

Similarly the E.M.F. triangle for the part BC is shown by BMC, which is drawn in the position shown because the point B in each triangle represents the one point B' between the two parts of the circuit, and therefore it should occupy only one position on the diagram. The total E.M.F. over the entire circuit is given then by AC, while the E.M.F. triangle for the entire circuit is found by completing the triangle AKC.

From this construction it is seen that the total resistance in the circuit is the sum of the resistances of each part; and the

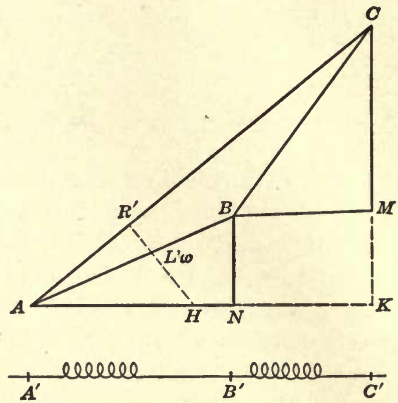

FIG. 95.—Two inductive circuits in series.

total inductance is the sum of the separate inductances. Of course the two parts are supposed to be far enough apart to avoid mutual induction between them. The angle of lag of the current behind the impressed E.M.F. is intermediate between the angles of lag in each part of the circuit considered separately, and is given by

$$a = \tan^{-1} \frac{L'w}{R'}.$$

Problems

1. Given two circuits in series, with $R_1 = 18$ ohms, $L_1 = 0.01$ henry, $R_2 = 4$ ohms, $L_2 = 0.02$ henry, $\omega = 400$. Find the E.M.F. necessary to

maintain 10 amperes through the circuit; also the E.M.F. over each part.

Solution.—Draw to scale a figure similar to Fig. 95. Then use then same scale to measure AC, AB, and BC.

2. Given the same circuit as above. What is the value of the current when the impressed E.M.F. is 100 volts?

Solution.—Draw the line AC to represent the value of E. At A construct the angle $CAH = \tan^{-1} \frac{L'\omega}{R'}$, as shown by the dotted lines, Fig. 95. Extend AH to meet the perpendicular from C at K. Then $I = AK/R'$, where $R' = R_1 + R_2$.

Solution.—Second method. Assume a value I' for the current and find the corresponding value E' for the impressed E.M.F. as above. Then $E' : 100 : : I' : I$.

3. When the impressed E.M.F. is 200 volts, what is the E.M.F. over each part of the above circuit?

Solution.—Draw the triangle KAC as before. Divide the side AK in the ratio of the two resistances, and the side KC in the ratio of the two inductances. Through the points M and N draw lines parallel to these sides; their intersection locates the position of B. The values of AB and BC can then be measured.

201. Inductive Circuits in Parallel.—In the case of a divided circuit having two or more inductances in parallel, it is much more difficult to calculate what part of the current will pass through each branch, and graphical methods become more useful. The following example for two inductive circuits in parallel can, of course, be extended to as many parallel circuits as desired.

Since each branch will have the same impressed E.M.F. the hypotenuse of each E.M.F. triangle will be identical. Let this be laid off to scale as shown by AB, Fig. 96. Since each triangle is right angled, it will be inscribed within a semicircle drawn on AB as a diameter. From the constants of the circuit the angle of lag in each branch can be determined. The base, AN, of the first triangle can then be laid off, making this angle with AB. The intersection of AN with the semicircle locates the other corner of this triangle at N, and the line NB completes the other side. The value of the current is $I_1 = AN/R_1$, and is laid off in the direction AN.

Similarly, the triangle AMB is laid out for the other branch,

and the value of the current determined. The resultant current is the geometrical sum, AI, of these two components, and it lags behind the impressed E.M.F. by the angle HAB.

The point H where the line AI cuts the semicircle is the right angled corner of a new triangle, AHB, which represents the resultant or equivalent effect of a single circuit which could replace the two parallel circuits. The E.M.F. AH is $R'I$, and HB is $L'\omega I$, where R' and L' denote the values of the resistance and the inductance of this equivalent circuit. These values can be taken from the figure as accurately as the lines can be measured.

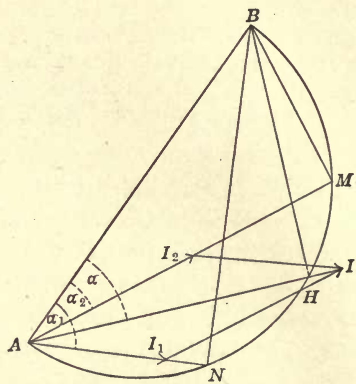

FIG. 96.—Inductive circuits in parallel.

Problem.—Given a coil with $R_1 = 22$ ohms and $L_1 = 0.03$ henry, in parallel with a second coil with $R_2 = 8$ ohms and $L_2 = 0.03$ henry. $E = 100$ volts, and $\omega = 400$. Find the values of the currents through each branch and in the main circuit; also the equivalent resistance and the equivalent inductance of the two coils in parallel, and the angle of lag of each current behind the impressed E.M.F.

202. Graphical Solution for Circuits having Capacity.—In a circuit having a condenser in series with a resistance and an alternating E.M.F. the relation at any instant is

$$e = Ri + \frac{q}{C}; \tag{1}$$

where $\frac{q}{C}$ is the difference of potential across the condenser of capacity C farads, when its charge is q coulombs.

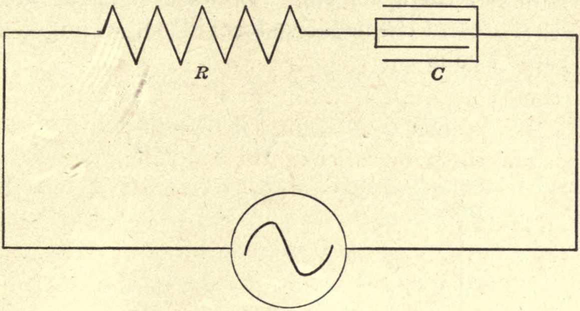

FIG. 97.

If the current flowing through R, Fig. 97, and into the condenser is following the sine law, its value at any instant is

$$i = I \sin \omega t \tag{2}$$

Then,

$$q = \int i dt = \int I \sin \omega t dt = -\frac{I}{\omega} \cos \omega t \tag{3}$$

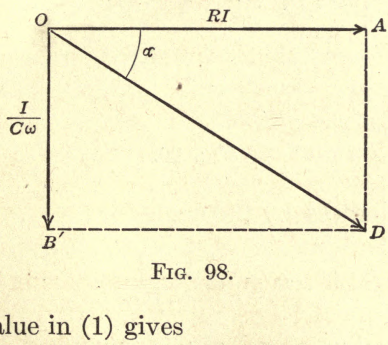

FIG. 98.

Putting this value in (1) gives

$$e = RI \sin \omega t - \frac{I}{C\omega} \cos \omega t \tag{4}$$

The two terms of this equation can be represented by two lines drawn at right angles, as was done in Article 194. The result will be different, however, for after laying off OA, Fig. 98, equal to RI, the other side, OB must be drawn *downward* in order to represent the negative term in (4). The geometrical sum of these two is

$$OD = E = I\sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

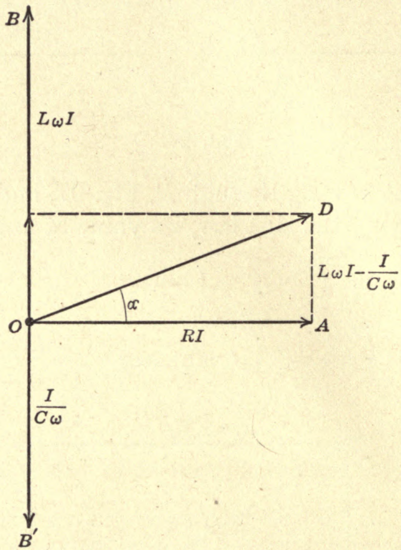

FIG. 99.

and this is behind OA by the angle

$$a = \tan^{-1}\frac{1}{RC\omega}$$

The current is thus ahead of the impressed E.M.F. by the angle a.

The resultant current for circuits in series and in parallel

can be obtained by combining figures like Fig. 98 as was done above for inductive circuits.

In case there is both capacity and inductance in the same circuit, the resultant E.M.F. required to maintain the current is found by combining Fig. 89 with Fig. 98, as shown in Fig. 99. The E.M.F's. $L\omega I$ and $\frac{I}{C\omega}$ are opposed to each other, and the algebraic sum of these two is combined with RI to give the resultant E.M.F., OD, Fig. 99.

203. Power Expended in an Alternating-current Circuit.— The power expended in a wire by an alternating current flowing through it is RI^2, where I denotes the mean square value of the current. If this same wire is wound into a coil having a large inductance, the amount of heat produced in the wire will be the same as before for the same current. But it will now require a greater E.M.F. to maintain the same current. However there is no expenditure of energy because of the inductance, since half of the time the induced E.M.F. opposes the current and the other half of the time it helps the current an equal amount.

The product RI^2 can be written as $RI \times I$, and RI is the E.M.F. necessary to maintain the current through the resistance of the circuit. Therefore the power expended is

$$W = RI \times I = EI \cos a$$

where $E \cos a$ is the component of the impressed E.M.F. that is in phase with the current. This factor, $\cos a$, is called the power factor of the circuit. Evidently the power factor becomes small for circuits having a large amount of inductance or of capacity.

This explains how a high E.M.F. accompanied by a large current in an inductive circuit may yet involve a small amount of power.

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